Week 1: solutions
ECON 100A

TA Allegra Saggese
Date September 29, 2025

Professor: Natalia Lazzati
Topic: Math review

Exercise problems are sourced primarily from Simon & Blume, *Mathematics for Economists* and Hoy et. al. *Mathematics for Economics (Vol 2)*, with changes to the numerics developed with ChatGPT. If you are interested in these textbooks, please let me know as you can borrow them from me for the quarter.

Problem 0

Solution 0: what is a Lagrange?

The Lagrangian function is often set up as:

$$L(x_1.x_2, \lambda) \equiv f(x_1, x_2) - \lambda(h(x_1, x_x) - c)$$

which represents a system of equation with three unknowns and two equations. A system of equations must have at least as many equations as free variables, and therefore with three unknowns we **need** three equations. In this case, our third equation is the constraint. Therefore, this Lagrangian is a way of writing a system of equations such that the constraint is equivalent to zero ($c - h(x_1, x_2) = 0$).

The Lagrangian allows us to reduce a constrained optimization problem in two variables to an unconstrained problem in three variables. The Lagrangian multiplier represents the **penalty** for the new constraint (and does have an economic interpretation such as a user fee or scarcity rents).

In order to get the solution of a Lagrangian we set all partial derivatives equal to zero and solve for the critical points.

$$\frac{\partial L}{\partial x_1} = 0$$

$$\frac{\partial L}{\partial x_2} = 0$$

$$\frac{\partial L}{\partial \lambda} = 0$$

Problem 1

Solution 1: first derivatives (univariate)

Find the derivatives of the following functions:

1. $-7x^3$

$$\frac{\partial f}{\partial x} = -21x^2$$

2. $\frac{(x-1)}{(x+1)}$

$$f(x) = \frac{(x-1)}{(x+1)} = \frac{u(x)}{v(x)}$$

$$\text{using the quotient rule: } \frac{u'(x)v(x) - u(x)v'(x)}{v(x)^2}$$

$$\frac{\partial f}{\partial x} = \frac{(1)(x+1) - (x-1)(1)}{(x+1)^2} \equiv \frac{2}{(x+1)^2}$$

3. $12x^{-2}$

$$\frac{\partial f}{\partial x} = -24x^{-3} \text{ or } -\frac{24}{x^3}$$

4. $4x^5 - 3x^{\frac{1}{2}}$

$$\frac{\partial f}{\partial x} = 20x^4 - 1.5x^{-\frac{1}{2}} \text{ or } 20x^4 - \frac{3}{2\sqrt{x}}$$

partial derivatives (x,y) Recall that to solve for a partial derivative, treat the other variable as if its a constant.

1. $4x^2y - 3xy^3 + 6x$

$$\frac{\partial f}{\partial x} = 8xy - 3xy^3 + 6$$
$$\frac{\partial f}{\partial y} = 4x^2 - 9xy^2 - 0$$

2. xy^2

$$\frac{\partial f}{\partial x} = y^2$$
$$\frac{\partial f}{\partial y} = 2xy$$

Problem 2

Solution 2: To get the solution of the system of equations, we are solving for an n-tuple of real numbers $(x_1, x_2, ...x_n)$ which satisfies each of the m equations. For example, we are often solving for (x, y) in a system of $m = 2$ equations. There are three methods of solving such systems:

- substitution: solve for one variable (x) in terms of the other variable (y). Substitute in the expression for for the other variable in that equation.
- elimination of variables: eliminate a variable by multiplying one equation by a number that allows for the elimination of one of the variables. For e.g.

$$x - 2y = 8 \tag{1}$$
$$3x + y = 3 \tag{2}$$
$$\rightarrow \text{multiple (1) by 3} \tag{3}$$

$$\begin{aligned} 3x - 6y &= 24 \\ 3x + y &= 3 \\ 0 - 7y &= 21 \Rightarrow y = 3 \end{aligned}$$

  then, use substitution to solve for x. This is also called Gaussian elimination (variable elimination then substitution).

- matrix methods: reduce the coefficient matrix down to a system of linear equations, equal to the given system, in which we can reduce to reduced row echelon form. This allows us to then get each pivot (x = 1, y = 1) equivalent to their numerical value.

$$1. \quad \Rightarrow \quad (x, y) = (2, 3)$$

$$2. \quad \Rightarrow \quad (x, y) = (17/9, 23/9)$$

$$3. \quad \Rightarrow \quad (x, y) = (4, 2)$$

Problem 3

Solution 3:

1. *take the derivative*
$$f'(x) = -2x + 6.$$
get FOCs (set equal to zero)
$$-2x + 6 = 0 \quad \Rightarrow \quad x = 3.$$
get SOCs
$$f''(x) = -2 < 0 \quad \Rightarrow \quad \text{local maximum at } x = 3.$$
check boundary
$$f(0) = 0, \quad f(3) = -9 + 18 = 9.$$
solution: $x^* = 3,\ f^* = 9.$

2. *gradient vector (vector of both partial derivatives)*
$$\nabla f(x, y) = (-2x + 4,\ -2y + 2).$$
set partial derivatives equal to zero (FOCs)
$$-2x + 4 = 0 \quad \Rightarrow \quad x = 2, \qquad -2y + 2 = 0 \quad \Rightarrow \quad y = 1.$$
can set up the Hessian matrix (matrix of second order partial derivatives)
$$H = \begin{bmatrix} \frac{\partial f}{\partial x^2} & \frac{\partial f}{\partial xy} \\ \frac{\partial f}{\partial xy} & \frac{\partial f}{\partial y^2} \end{bmatrix}$$
$$H = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix},$$
which is negative definite (found by using the determinant formula such that the first principle minor is -2 (given), and the second principle minor is$(-2 * -2) - 0 = 4$, so $(2, 1)$ is a maximum because the signs in the matrix alternative negative, positive.
evaluate function at candidate values:
$$f(2, 1) = -(2^2) - (1^2) + 4(2) + 2(1) - 5 = 0.$$
solution $(x^*, y^*) = (2, 1),\ f^* = 0.$

Problem 4

Solution 4: using Lagrange

$$\mathcal{L}(x, y, \lambda) = xy + \lambda(10 - 2x - y).$$

FOCs:

$$\frac{\partial \mathcal{L}}{\partial x} : \; y - 2\lambda = 0, \qquad \frac{\partial \mathcal{L}}{\partial y} : \; x - \lambda = 0, \qquad 2x + y = 10.$$

From the first two conditions:

$$\lambda = x, \qquad y = 2x.$$

Substitute the value of λ into the constraint:

$$2x + 2x = 10 \;\; \Rightarrow \;\; x = 2.5, \quad y = 5.$$

Checking the boundary conditions: If $x = 0$ or $y = 0$, then $u(x, y) = 0$. An interior solution gives:

$$u(2.5, 5) = 12.5,$$

which is larger than the value of the boundary solutions.
solution:

$$(x^*, y^*) = (2.5, 5), \quad u^* = 12.5.$$

Week 2: solutions
ECON 100A

TA Allegra Saggese
Date October 5, 2025

Professor: Natalia Lazzati
Topic: consumer problem

Exercise problems are sourced primarily from Varian textbook, with changes to the numerics developed with ChatGPT.

Problem 0

Q: What is the relationship between a utility function and indifference curves?

Solution 0: Utility and indifference curves

A utility function will assign the same number (the total *utility* from that bundle) to all bundles that lie on a single indifference curve. So, any bundle, represented as an (x, y) point, along a single indifference curve has the same utility. As indifference curves move outward from the origin $(0, 0)$ - higher - the utility value increases.

Problem 1

Q: If the price of good 1 doubles and the price of good 2 triples, does the budget line become flatter or steeper?

Solution 1: Price changes

We know $p_1 \to 2p_1$ and $p_2 \to 3p_2$. We know the budget line is given as the negative ratio of p_1 and p_2. As such,

$$\text{budget line} \to -\frac{p_1}{p_2} \Rightarrow -\frac{2p_1}{3p_2}$$

So the budget line is being multiplied by a scalar equal to $\frac{2}{3}$. This decreases the absolute value of the slope and the budget line becomes **flatter**.

Problem 2

Solution 2: Compare bundles

1. Check affordability of purchasing the bundle $(25, 10)$. You can do this by simply plugging in the quantity from the given bundle and the price (as given) into the budget constraint. Check against your total income, m:

$$p_1x_1 + p_2x_2 = 3(25) + 6(10) = 75 + 60 = 135 > 120$$

$\Rightarrow$ **not affordable**

2. After a specific tax of \$3 on good 1, we can calculate the new price p_1, while p_2 remains the same:

$$p_1' = 3 + t = 3 + 3 = 6, \quad p_2 = 6$$

The new budget equation becomes:

$$6x_1 + 6x_2 = 120$$

3. Check affordability of the bundles $(25, 10)$ either by plugging in (as below):

$$6(25) + 6(10) = 150 + 60 = 210 > 120$$

or noticing that if the bundle was not affordable under the original budget constraint, there is no way that the bundle will be affordable with a price increase.

$\Rightarrow$ **not affordable**

4. If the consumer buys $(12, 8)$ at the new prices $(p_1, p_2) = (6, 6)$, then we can compare the total spend at these new prices with the prices at the no-tax cost $(p_1, p_2) = (3, 6)$:

$$\begin{aligned} p_1x_1 + p_2x_2 = 6(12) + 6(8) \qquad &= 120 \\ p_1x_1 + p_2x_2 = 3(12) + 6(8) \qquad &= 84 \\ &= 36 \end{aligned}$$

Or you can multiply the tax (\$3) on the quantity of good 1 purchases (12) to get that the total tax revenue collected is \$36.

$$t \cdot x_1 = 3(12) = 36$$

Problem 3

Recall the utility as given: $u(x_1, x_2) = x_1^2 x_2^5$

Solution 3: Marginal utility

1. To get the marginal utility with respect to a good, you need to take the **partial derivative** of the utility function with respect to that specific good (in this case, either good 1 or good 2):

$$MU_1 = \frac{\partial u}{\partial x_1} = 2x_1 x_2^5 \qquad MU_2 = \frac{\partial u}{\partial x_2} = 5x_1^2 x_2^4$$

2. The marginal rate of substitution (MRS) is simply the ratio of the marginal utilities between the two goods. It tells you how much utility you get in terms of one unit of utility of the other good.

$$\mathrm{MRS}_{1,2} = \left|\frac{MU_1}{MU_2}\right| = \frac{2x_1 x_2^5}{5x_1^2 x_2^4} = \boxed{\frac{2x_2}{5x_1}}$$

Problem 4

Solution 4: Marginal utility

Consumer A: $u(x_1, x_2) = (x_1 + 2x_2)^2$
Consumer B: $v(x_1, x_2) = x_1 + 2x_2$

1. Marginal utilities:

   Consumer A:

$$MU_1^A = \frac{\partial u}{\partial x_1} = 2(x_1 + 2x_2)$$

$$MU_2^A = \frac{\partial u}{\partial x_2} = 4(x_1 + 2x_2)$$

$$\mathrm{MRS}_{1,2}^A = \frac{MU_1^A}{MU_2^A} = \frac{1}{2}$$

   Consumer B:

$$MU_1^B = \frac{\partial u}{\partial x_1} = 1, \qquad MU_2^B = \frac{\partial u}{\partial x_2} = 2$$

$$\mathrm{MRS}_{1,2}^B = \frac{MU_1^B}{MU_2^B} = \frac{1}{2}$$

2. Since $\mathrm{MRS}_{1,2}^A = \mathrm{MRS}_{1,2}^B = \frac{1}{2}$ and we can rewrite the utility functions for consumer A and B as the following:

$$u(x_1, x_2) = [v(x_1, x_2)]^2,$$

   Then we know, mathematically, the transformation $t \mapsto t^2$ is strictly increasing for $t > 0$. Because of this, both utility functions represent the same preferences just on different indifference curves.

Problem 5

Solution 5: The Fun Index

Recall the given definitions: $A = P + S + C$, $B = 3P + 2S + 4C$, N visits, $\mathrm{FI} = \dfrac{A\,B}{N}$.

(a) Compute A, B, and FI for each friend

Friend	N	P	S	C	A	B	$\mathrm{FI} = AB/N$
Apollo	10	6	8	3	17	46	$\dfrac{17 \cdot 46}{10} = 78.2$
Vesta	12	8	6	5	19	56	$\dfrac{19 \cdot 56}{12} = 88.\overline{6}$
Jupiter	9	5	10	2	17	43	$\dfrac{17 \cdot 43}{9} = 81.\overline{2}$
Edesia	11	7	7	7	21	63	$\dfrac{21 \cdot 63}{11} = 120.2727\ldots$

(b) Rank

Based on the numeric values in the table above, we can see that Edesia has the highest fun index. From there we rank on the value:

Edesia > Vesta > Jupiter > Apollo

(c) Indifference curves in (A, B) space

Let's assume a constant Fun Index. We will call $\mathrm{FI} = \mathrm{FI} = \bar{r}$, and keep fixed N:

$$\frac{A\,B}{N} = \bar{r} \iff B = \frac{N\bar{r}}{A}.$$

Through each friend's point (A_0, B_0), use $N\bar{r} = A_0 B_0$:

$$\text{Apollo: } B = \frac{782}{A}, \quad \text{Vesta: } B = \frac{1064}{A}, \quad \text{Jupiter: } B = \frac{731}{A}, \quad \text{Edesia: } B = \frac{1323}{A}.$$

Week 3: solutions
ECON 100A

TA Allegra Saggese
Date October 10, 2025
Professor: Natalia Lazzati
Topic: midterm 1 review

Exercise problems are sourced primarily from Varian textbook, with changes to the numerics developed with ChatGPT.

Problem 0

Q: Derive the demand and conduct comparative statistics on $U(I, kW) = \min\{I,\, kW\}$

Important to recall: Utility is given by $U(I, kW) = \min\{I,\, kW\}$, where I is insulation and W is wood. Prices are p_I and p_W, and income is m. The budget constraint is:

$$p_I I + p_W W \leq m \qquad I, W \geq 0$$

Solution 0: A warm house

(a) Deriving demand functions. Because heat and protection are perfect complements, the consumer will always choose $I = kW$. Let x denote the common level of utility (heat and protection) actually consumed:

$$x = I = kW \quad \Rightarrow \quad I = x,\ W = \frac{x}{k}.$$

The total expenditure required to achieve level x is

$$p_I I + p_W W = p_I x + p_W \frac{x}{k} \equiv x\Big(p_I + \frac{p_W}{k}\Big).$$

Setting this RHS of the equation equal to income m gives:

$$x^* = \frac{m}{p_I + \frac{p_W}{k}}.$$

Thus, the optimal quantities are

$$I^* = x^* = \frac{m}{p_I + \frac{p_W}{k}}, \qquad W^* = \frac{x^*}{k} = \frac{m}{kp_I + p_W}.$$

Note that $I^* = kW^*$ which is consistent with perfect complementarity.

(b) Comparative statics with respect to k. From the expression for W^*,

$$W^*(k) = \frac{m}{kp_I + p_W}.$$

Taking the derivative,

$$\frac{\partial W^*}{\partial k} = -\frac{mp_I}{(kp_I + p_W)^2} < 0 \quad \text{for } p_I > 0.$$

Hence, as k increases (each unit of wood generates more heat), the consumer needs less wood. Intuitively, the more efficient wood is (larger k is), the rate of exchange between wood and insulation decreases, as less wood is needed to meet the same heat as insulation.

(c) Perfect substitutes case. If heat and protection were perfect substitutes, we write the utility function as: $U(I, kW) = I + kW$.

Here the consumer will allocate all spending to the cheaper source of "effective protection." One unit of protection costs p_I through insulation and p_W/k through wood. Thus,

$$(I^*, W^*) = \begin{cases} \left(\frac{m}{p_I}, 0\right), & \text{if } p_I < \frac{p_W}{k} \quad \text{(all insulation)},\\ \left(0, \frac{m}{p_W}\right), & \text{if } p_I > \frac{p_W}{k} \quad \text{(all wood)},\\ \text{any mix s.t. } p_I I + p_W W = m, & \text{if } p_I = \frac{p_W}{k} \quad \text{(indifferent)}. \end{cases}$$

As k increases (making wood more effective), the relative cost p_W/k falls. Eventually, if $p_W/k < p_I$, the consumer switches entirely to wood.

Problem 1

Q: Are the two choices made by the consumer consistent with utility maximization?

Solution 1: Consistency

A consumer buys two goods, x_1 and x_2. Initially, prices are $(p_1, p_2) = (15, 10)$ and the consumer chooses basket 1:

$$(x_1, x_2) = (10, 3).$$

Later, prices change to $(p_1, p_2) = (12, 12)$, and the consumer chooses basket 2:

$$(x_1, x_2) = (5, 10).$$

Question: Are these two choices consistent with utility maximization?

Solution. We apply the *Weak Axiom of Revealed Preference (WARP)*. Let the consumer's income in each period be such that both chosen bundles are affordable in their respective periods.
Under the first prices $(15, 10)$, the cost of each bundle is:

$$\text{Cost of bundle 1: } 15(10) + 10(3) = 150 + 30 = 180,$$

$$\text{Cost of bundle 2: } 15(5) + 10(10) = 75 + 100 = 175.$$

Since bundle 2 costs only $175 < 180$, it was *affordable* when bundle 1 was chosen. Because the consumer chose bundle 1 instead of bundle 2 when both were affordable, bundle 1 is *revealed preferred* to bundle 2.
Now consider the second price vector $(12, 12)$:

$$\text{Cost of bundle 2: } 12(5) + 12(10) = 60 + 120 = 180,$$

$$\text{Cost of bundle 1: } 12(10) + 12(3) = 120 + 36 = 156.$$

Here bundle 1 costs $156 < 180$, so it was affordable when the consumer chose bundle 2. Thus, in the second situation, the consumer chose bundle 2 even though bundle 1 was cheaper and previously revealed preferred. This violates the Weak Axiom of Revealed Preference.

Conclusion: The two choices are *not consistent* with utility maximization, because the consumer's behavior violates revealed preference theory.

Problem 2

Q: Can you explain why taking a monotonic transformation of a utility function does not change the marginal rate of substitution (MRS)?

Solution 2: Monotonicity and the MRS

Let a consumer have a utility function $u(x_1, x_2)$ representing preferences over two goods. The marginal rate of substitution (MRS) between goods x_1 and x_2 is defined as

$$MRS_{12} = \frac{MU_1}{MU_2} = \frac{\partial u/\partial x_1}{\partial u/\partial x_2},$$

which measures the rate at which the consumer is willing to trade good 2 for good 1 while remaining on the same indifference curve.

Now consider a *monotonic transformation* of this utility function:

$$v(x_1, x_2) = f(u(x_1, x_2)),$$

where $f(\cdot)$ is a strictly increasing, differentiable function. Because f is monotonic, v represents the same preference ordering as u. We can now compute the marginal utilities under the transformed utility function v:

$$\frac{\partial v}{\partial x_1} = f'(u(x_1, x_2)) \cdot \frac{\partial u}{\partial x_1}, \qquad \frac{\partial v}{\partial x_2} = f'(u(x_1, x_2)) \cdot \frac{\partial u}{\partial x_2}.$$

Thus, the new MRS is

$$MRS_{12}^{(v)} = \frac{\frac{\partial v}{\partial x_1}}{\frac{\partial v}{\partial x_2}} = \frac{f'(u)\,\frac{\partial u}{\partial x_1}}{f'(u)\,\frac{\partial u}{\partial x_2}} = \frac{\frac{\partial u}{\partial x_1}}{\frac{\partial u}{\partial x_2}} = MRS_{12}^{(u)}.$$

Intuition: A monotonic transformation $f(u)$ rescales utility levels but preserves the shape of indifference curves, since f only changes the numerical labels assigned to each utility level, not the underlying ranking. Because the *slope* of an indifference curve is the MRS, and monotonic transformations leave these slopes unchanged, the consumer's marginal rate of substitution—and therefore all choices derived from utility maximization—remain the same.

$$\boxed{\text{Monotonic transformations preserve the MRS (and hence the same preferences).}}$$

Week 5: solutions
ECON 100A

TA Allegra Saggese
Date October 29, 2025

Professor: Natalia Lazzati
Topic: WARP and revealed preferences

Exercise problems are sourced primarily from Varian textbook, with changes to the numerics developed with ChatGPT.

Problem 0

Solution 0: WARP

$$\text{Year 1 prices } (p_1, p_2) = (100, 100), \quad x^{(1)} = (100, 100),$$
$$\text{Year 2 prices } (p_1, p_2) = (100, 80), \quad x^{(2)} = (120, m).$$

Incomes should be calculated such that you exhaust the total budget, meaning you should plug in the given bundles, at their prices, and find out how much income it would cost:

$$I_1 = 100 \cdot 100 + 100 \cdot 100 = 20{,}000$$
$$I_2 = 100 \cdot 120 + 80 \cdot m = 12{,}000 + 80m$$

Then we want to do an affordability cross–checks. This means we want to evaluate the cost of the bundle in year 1 *at year 2 prices*, and vice versa:

$$\text{Cost of } x^{(2)} \text{ at Year 1 prices } = 100 \cdot 120 + 100 \cdot m = 12{,}000 + 100m,$$
$$x^{(2)} \text{ affordable in Year 1 } \iff 12{,}000 + 100m \leq 20{,}000 \iff m \leq 80.$$

$$\text{Cost of } x^{(1)} \text{ at Year 2 prices } = 100 \cdot 100 + 80 \cdot 100 = 18{,}000,$$
$$x^{(1)} \text{ affordable in Year 2 } \iff 18{,}000 \leq 12{,}000 + 80m \iff m \geq 75.$$

(a) Inconsistency with WARP: Recall inconsistency with WARP means that you would be willing (and able, i.e. not constrained by the budget) to purchase the same goods in both periods. So, if you buy a *different* bundle in year 2 from year 1, even though you could afford the bundle from year 1 during year 2, you are inconsistent in your preferences.

$$m > 75 \Rightarrow x^{(1)} \text{ is affordable when } x^{(2)} \text{ is chosen } \Rightarrow x^{(2)} \succ_R x^{(1)}.$$
$$m < 80 \Rightarrow x^{(2)} \text{ is affordable when } x^{(1)} \text{ is chosen } \Rightarrow x^{(1)} \succ_R x^{(2)}.$$

Both strict revealed preferences occur *simultaneously* when

$$\boxed{75 < m < 80}$$

so WARP is violated exactly on $(75, 80)$. (At the boundaries $m = 75$ or $m = 80$ one direction is only weak, hence no WARP violation.)

(b) When is Year 1 revealed preferred to Year 2?

$$m \leq 80 \implies \text{Year 2 bundle affordable at Year 1 prices} \implies x^{(1)} \succeq_R x^{(2)}.$$

Thus

$$\boxed{m \leq 80 \Rightarrow \text{ Year 1 is revealed preferred to Year 2.}}$$

Problem 0

Solution 1: WARP

$$\text{Year 1: } (p_1, p_2) = (10, 10),\ y = 2000,\ x^{(1)} = (50, 150),$$
$$\text{Year 2: } (p_1, p_2) = (15, 5),\ y = 2000,\ x^{(2)} = (M, N).$$

Budgets are exhausted each year. But we look here and plug in (M,N) into the budgets in order to evaluate.

$$\text{Year 1 spend: } 10 \cdot 50 + 10 \cdot 150 = 2000,$$
$$\text{Year 2 budget line: } 15M + 5N = 2000.$$

Again, like in problem 0, we assess the cross–price affordability of the bundles:

$$\text{Cost of } x^{(2)} \text{ at Year 1 prices } = 10M + 10N = 10(M + N).$$
$$x^{(2)} \text{ affordable in Year 1 } \iff 10(M + N) \le 2000 \iff M + N \le 200.$$

$$\text{Cost of } x^{(1)} \text{ at Year 2 prices } = 15 \cdot 50 + 5 \cdot 150 = 750 + 750 = 1500\ (< 2000).$$

Hence $x^{(1)}$ is *always* affordable in Year 2.

(a) All (M, N) consistent with WARP.

- In Year 2, since $x^{(1)}$ is affordable and the consumer chose $x^{(2)}$,

$$x^{(2)} \succ_R x^{(1)} \quad \text{(strict revealed preference)}$$

- WARP is violated if *also* $x^{(1)} \succeq_R x^{(2)}$ at Year 1 prices (with strict in at least one direction). That happens whenever $M + N \le 200$.
- To avoid contradiction: $M + N > 200$ must hold. Then, with the Year 2 budget $15M + 5N = 2000$ and $M, N \ge 0$.

Set of (M, N) that is consistent with WARP is:

$$\boxed{\{(M, N) \in R^2_+ :\ 15M + 5N = 2000,\quad M + N > 200\}}.$$

(b) Comparing $(50, 150)$ with any WARP–consistent (M', N'). From above:

$$x^{(1)} \text{ is affordable in Year 2, and the choice was } x^{(2)} = (M', N').$$
$$\text{So we know, } x^{(2)} \succ_R x^{(1)} \text{ by revealed preference}$$

$$\to \text{The consumer prefers } (M', N') \text{ to } (50, 150)$$

This is the revealed preference, such taht Year 1 does not overturn this since $M' + N' > 200$ makes $x^{(2)}$ *not* affordable at Year 1 prices.

Week 6: solutions
ECON 100A

TA Allegra Saggese
Date November 3, 2025

Professor: Natalia Lazzati
Topic: Intertemporality

Exercise problems are sourced primarily from Varian textbook, with changes to the numerics developed with ChatGPT.

Problem 0

Solution 0: Intuition

Let c_1 be consumption today and c_2 consumption tomorrow, with income (m_1, m_2) and interest rate r. Can draw the budget line with c_2 on the vertical axis and c_1 on the horizontal axis is

$$c_2 = (1 + r)(m_1 - c_1) + m_2$$

Its slope is

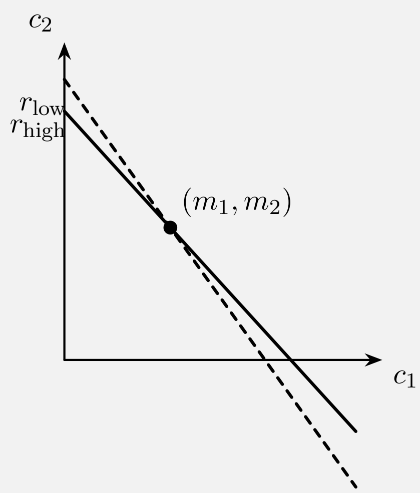
$$\frac{\partial c_2}{\partial c_1} = -(1 + r)$$

From this, we see as r rises, the line becomes *steeper* in (c_1, c_2) space. We can map out an example graphically, starting with end points:

$$\text{when } c_1 = 0 : \quad c_2 = (1 + r)m_1 + m_2$$

$$\text{when } c_2 = 0 : \quad c_1 = m_1 + \frac{m_2}{1 + r}$$

The budget line pivots around the endowment point (m_1, m_2), becoming steeper as r increases.



Problem 1

Solution 1: Better off?

Let c_1 be today's consumption (horizontal axis) and c_2 tomorrow's consumption (vertical axis). Endowment (m_1, m_2), interest rate r. The budget line with c_2 on the vertical axis is

$$c_2 = (1+r)(m_1 - c_1) + m_2$$

with slope $\dfrac{dc_2}{dc_1} = -(1+r)$ A *decrease* in r makes the line *flatter*.

What happens to welfare when r falls?

If the consumer remains a lender $(c_1^* < m_1)$: worse off

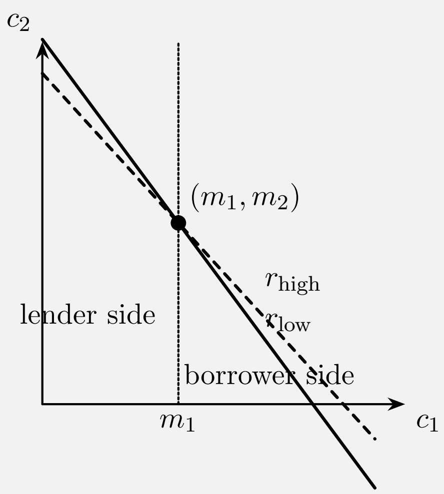
Reason: the return to saving falls, shrinking opportunities on the lender side

If the consumer becomes a borrower $(c_1^* > m_1)$: better off

Reason: borrowing is cheaper, the budget set expands on the borrower side

If the consumer stays at the endowment $(c_1^* = m_1)$: unaffected

These statements hold for standard monotone, convex preferences: the pivot makes the feasible set worse on the saving side and better on the borrowing side relative to the endowment (income). We can also think about what happens to *income*, and what happens to consumption *decisions*. When r falls, the relative price of future consumption rises ($p_{c_2} = 1/(1+r) \uparrow$), so a consumer will be more willing to make purchases today, tilting demand toward more c_1 and less c_2. For a net lender, lower interest income will occur, making them worse off. For a net borrower, there is a positive effect from paying less interest, making them better off.



Problem 2

Solution 2: Optimal bundle *Note: This problem is based off of problem 2 on problem set 5.*

Start by setting up the budget constraint in present value

$$c_1 + \frac{c_2}{1+r} = m_1 + \frac{m_2}{1+r}$$

$$c_1 + \frac{c_2}{1.2} = 18{,}000 + \frac{24{,}000}{1.2} = 18{,}000 + 20{,}000 = 38{,}000$$

Take the first order conditions, and set MRS = relative price

$$MRS = \frac{\partial U/\partial c_1}{\partial U/\partial c_2} = \frac{c_2}{c_1}$$

$$\frac{c_2}{c_1} = 1 + r = 1.2 \;\Rightarrow\; c_2 = 1.2\, c_1$$

Eliminate one of the free variables (c_2) and plug into the budget constraint:

$$c_1 + \frac{1.2c_1}{1.2} = 38{,}000 \;\Rightarrow\; 2c_1 = 38{,}000 \;\Rightarrow\; c_1^* = 19{,}000$$

$$c_2^* = 1.2\, c_1^* = 1.2 \times 19{,}000 = 22{,}800$$

Calculate the saving to see if the consumer is a saver (positive) or a borrower (negative)

$$s = m_1 - c_1^* = 18{,}000 - 19{,}000 = -1{,}000$$

Negative saving ⇒ **borrower**

Problem 3

Solution 3: Opportunity cost

(a) To calculate the explicit cost we take the cost, per year. It is only a one-year graduate program, so in total:

$$\text{Explicit cost of enrolling} = \$30{,}000$$

(b) Opportunity cost of enrolling includes both the monetary cost, and the additional lost income from not working.

$$\text{Monetary opportunity cost} = \text{foregone wage this year} = \$50{,}000$$

(c) Present value comparison of the two choices:
Let $r = 0.05$. Compare Work-now vs Study-now over a two period horizon, such that scenario one, *Work-now* is the present value of working two years, while scenario two, *Study-now* is the present value of going to graduate school and working one year.

$$\text{PV(Work-now)} = 50{,}000 + \frac{50{,}000}{1+r}$$

$$\text{PV(Study-now)} = -30{,}000 + \frac{70{,}000}{1+r}$$

Net advantage of studying

$$\Delta PV \equiv \text{PV(Study-now)} - \text{PV(Work-now)} = \big(-30{,}000 - 50{,}000\big) + \frac{70{,}000 - 50{,}000}{1+r}$$

$$\Delta PV = -80{,}000 + \frac{20{,}000}{1.05}$$

Over only two years, $\Delta PV < 0$. Therefore, if evaluating against a two-year time horizon, she should **not go to graduate school**. Over a longer career horizon with many years at the higher wage, ΔPV may become positive. In the short run, though, it is costly, given the opportunity cost of the graduate program is greater than the discounted difference in earnings in the first year of work. But you could see how the sum of discounted earnings over a longer horizon, let's say 20 years, could increase her present value of graduate school.

Week 7: solutions
ECON 100A

TA Allegra Saggese
Date November 6, 2025

Professor: Natalia Lazzati
Topic: midterm 2 review

Exercise problems are sourced primarily from Varian textbook and course materials from ECON204A (graduate microeconomics) with changes to the numerics developed with ChatGPT.

Problem 1

Solution 1a: Intertemporal choice

1. The budget restriction in terms of future value is

$$c_1(1.1) + c_2 = 100 \times (1.1) + 200 = 310$$

2. The consumer's problem is

$$\max_{c_1,c_2}\{U(c_1,c_2) = c_1 + 2c_2\} \quad \text{subject to} \quad c_1(1.1) + c_2 = 310$$

Marginal utilities are

$$MU_1 = 1, \qquad MU_2 = 2$$

Compare marginal utility per (future-value) price

$$\frac{MU_1}{1.1} = \frac{1}{1.1} < \frac{MU_2}{1} = \frac{2}{1}$$

Since the marginal payoff from an extra unit of c_2 exceeds that of c_1 at the margin, the optimum is a corner with all resources devoted to period 2 consumption:

$$c_1^* = 0, \qquad c_2^* = 310$$

3. After the tax of \$20 on period-2 income we have $m_2' = 180$. The future-value budget becomes

$$c_1(1.1) + c_2 = 100 \times (1.1) + 180 = 290$$

The consumer's problem is

$$\max_{c_1,c_2}\{c_1 + 2c_2\} \quad \text{subject to} \quad c_1(1.1) + c_2 = 290$$

The marginal utilities are unchanged, and the comparison again gives

$$\frac{MU_1}{1.1} = \frac{1}{1.1} < \frac{MU_2}{1}$$

Hence the post-tax optimum remains a corner with

$$c_1^* = 0, \qquad c_2^* = 290$$

Solution 1b: Intertemporal choice

Anna's utility is $U(c_1, c_2) = c_1 + c_2$. Prices are \$1 per loaf in both periods, $r = 0.20$, $m_1 = 2000$ and $m_2 = 1200$.

1. **Budget constraint (future-value form)**

   The future-value form of the intertemporal budget constraint is

   $$(1 + r)c_1 + c_2 = (1 + r)m_1 + m_2$$

   With $r = 0.20$ this becomes

   $$1.2\,c_1 + c_2 = 1.2 \times 2000 + 1200 = 3600$$

2. **Optimal consumption and savings**

   Given $U(c_1, c_2) = c_1 + c_2$, Anna is indifferent between consuming today and tomorrow in utility units, so she will choose a corner solution (all consumption in one period). Compare the two corner allocations.

   If she consumes everything in period 1, then period-1 consumption equals period-1 income plus the present value of period-2 income converted to period 1, i.e.

   $$c_1 = m_1 + \frac{m_2}{1 + r} = 2000 + \frac{1200}{1.2} = 3000$$

   If she consumes everything in period 2, then period-2 consumption equals the future value of lifetime resources, i.e.

   $$c_2 = (1 + r)m_1 + m_2 = 3600$$

   Because $3600 > 3000$ she prefers to consume entirely in period 2. Therefore the optimum is

   $$c_1^* = 0, \qquad c_2^* = 3600$$

   Savings in period 1 are

   $$s = m_1 - c_1^* = 2000$$

3. **Budget constraint with taxes**

   Now suppose Anna faces an income tax of \$100 in period 1 and \$200 in period 2. Then after-tax incomes are

   $$m_1' = 2000 - 100 = 1900, \qquad m_2' = 1200 - 200 = 1000$$

   The future-value budget is

   $$1.2\,c_1 + c_2 = 1.2 \times 1900 + 1000 = 3280$$

Problem 2

Solution 2: Revelead preferences

$$\text{Year 1 prices } (p_1, p_2) = (100, 100), \quad x^{(1)} = (100, 100),$$
$$\text{Year 2 prices } (p_1, p_2) = (100, 80), \quad x^{(2)} = (120, m).$$

Incomes should be calculated such that you exhaust the total budget, meaning you should plug in the given bundles, at their prices, and find out how much income it would cost:

$$I_1 = 100 \cdot 100 + 100 \cdot 100 = 20{,}000$$
$$I_2 = 100 \cdot 120 + 80 \cdot m = 12{,}000 + 80m$$

Then we want to do an affordability cross–checks. This means we want to evaluate the cost of the bundle in year 1 *at year 2 prices*, and vice versa:

$$\text{Cost of } x^{(2)} \text{ at Year 1 prices } = 100 \cdot 120 + 100 \cdot m = 12{,}000 + 100m,$$
$$x^{(2)} \text{ affordable in Year 1 } \iff 12{,}000 + 100m \leq 20{,}000 \iff m \leq 80.$$

$$\text{Cost of } x^{(1)} \text{ at Year 2 prices } = 100 \cdot 100 + 80 \cdot 100 = 18{,}000,$$
$$x^{(1)} \text{ affordable in Year 2 } \iff 18{,}000 \leq 12{,}000 + 80m \iff m \geq 75.$$

(a) Inconsistency with WARP: Recall inconsistency with WARP means that you would be willing (and able, i.e. not constrained by the budget) to purchase the same goods in both periods. So, if you buy a *different* bundle in year 2 from year 1, even though you could afford the bundle from year 1 during year 2, you are inconsistent in your preferences.

$$m > 75 \Rightarrow x^{(1)} \text{ is affordable when } x^{(2)} \text{ is chosen } \Rightarrow x^{(2)} \succ_R x^{(1)}.$$
$$m < 80 \Rightarrow x^{(2)} \text{ is affordable when } x^{(1)} \text{ is chosen } \Rightarrow x^{(1)} \succ_R x^{(2)}.$$

Both strict revealed preferences occur *simultaneously* when

$$\boxed{75 < m < 80}$$

so WARP is violated exactly on $(75, 80)$. (At the boundaries $m = 75$ or $m = 80$ one direction is only weak, hence no WARP violation.)

(b) When is Year 1 revealed preferred to Year 2?

$$m \leq 80 \implies \text{Year 2 bundle affordable at Year 1 prices} \implies x^{(1)} \succeq_R x^{(2)}.$$

Thus

$$\boxed{m \leq 80 \ \Rightarrow \ \text{Year 1 is revealed preferred to Year 2.}}$$

Problem 3

Solution 3: Consumer review

Suppose $p_1 = 5$, $p_2 = 3$, $m = 150$ and

$$U(x_1, x_2) = x_1^{1/4} x_2^{3/4}.$$

(a) Marginal utilities and MRS

$$\frac{\partial U}{\partial x_1} = \tfrac{1}{4}\, x_1^{-3/4} x_2^{3/4} \qquad \frac{\partial U}{\partial x_2} = \tfrac{3}{4}\, x_1^{1/4} x_2^{-1/4}$$

The marginal rate of substitution MRS_{12} is

$$MRS_{12} = \frac{\partial U/\partial x_1}{\partial U/\partial x_2} = \frac{\tfrac{1}{4}\, x_1^{-3/4} x_2^{3/4}}{\tfrac{3}{4}\, x_1^{1/4} x_2^{-1/4}} = \frac{1}{3}\,\frac{x_2}{x_1}$$

(b) Optimal consumption

At an interior optimum the tangency condition requires

$$MRS_{12} = \frac{p_1}{p_2}$$

so

$$\frac{1}{3}\,\frac{x_2}{x_1} = \frac{p_1}{p_2} \quad \Rightarrow \quad \frac{x_2}{x_1} = 3\frac{p_1}{p_2}$$

With $p_1/p_2 = 5/3$ we get

$$\frac{x_2}{x_1} = 3 \cdot \frac{5}{3} = 5 \quad \Rightarrow \quad x_2 = 5x_1$$

Use the budget constraint

$$p_1 x_1 + p_2 x_2 = m$$

substitute $x_2 = 5x_1$

$$5x_1 + 3(5x_1) = 150 \quad \Rightarrow \quad 20x_1 = 150 \quad \Rightarrow \quad x_1^* = 7.5$$

$$x_2^* = 5 \times 7.5 = 37.5$$

(c) Log utility case

If

$$U(x_1, x_2) = \tfrac{1}{4}\ln x_1 + \tfrac{3}{4}\ln x_2$$

then the Cobb–Douglas expenditure shares are unchanged: the consumer spends fraction 1/4 of income on good 1 and fraction 3/4 on good 2. Hence

$$x_1^* = \frac{(1/4)\, m}{p_1} = \frac{0.25 \times 150}{5} = 7.5$$

$$x_2^* = \frac{(3/4)\, m}{p_2} = \frac{0.75 \times 150}{3} = 37.5$$

Thus the optimal demands coincide with the answers above.

Week 8: solutions
ECON 100A

TA Allegra Saggese
Date November 17, 2025

Professor: Natalia Lazzati
Topic: Firms - monopolies

Exercise problems are sourced primarily from Varian textbook and course materials from ECON204A (graduate microeconomics) with changes to the numerics developed with Chat-GPT. Additional materials were taken from Penn State teaching materials.[1]

[1] https://courses.ems.psu.edu/ebf200/node/139

Problem 1

Solution 1.1: What are monopolies?

To identify monopolies, let's think of some firms that may currently be operating in violations of the normal rules of a competitive market. These violations, need to be measured. Normally we assess the following:

- **Market share**: what portion of total goods in a sector, let's say agriculture products, are sold by that one company?
- Is the firm a **price setter**? Can they change their price (up or down) and still maintain control of their market power?
- Does the firm engage in **anti-competitive or unfair strategies** (i.e. buying competitors via merger or acquisition, selling at a loss temporarily, overcharging consumers, or stifling innovation)?[a]
- Is there a **significant barrier to entry**? If so, is an existing player blocking others from entering?

Currently, the US has a number of ongoing litigation suits open with US companies, including Amazon and Meta - two large technology firms. They both have been accused of undertaking unfair or strategic anti-competitive moves to ensure market dominance. These cases are pending, and as thus *are not clear examples of monopolies*, although the government has accused them of being so based on evidence gathered by the US Department of Justice and the Federal Trade Commission (FTC).

While still a pending case, currently the FTC has sued three pharmaceutical benefits manager (PBMs) for what they claim is unfair pricing coordinating, and in a sense creating an *oligopoly* structure in the market.[b] The FTC, under its current case, has managed to reduce the cost of inhalers (medical device for asthma from $ 500 or more to just $35.[c]. This is because they identified collusion between three large firms, where they agreed to inflate prices in an effort to artificially boost profits.

Now, why is this bad? Maybe from a moral - normative - standpoint, we don't like the behavior. But in economics, we look at the positive implications. Here, we might actually see a **net welfare loss** where consumers pay more than the marginal cost of the goods, and thus have less money to spend on other things - like investing in their businesses, buying assets, or buying other consumer goods.

- **Normative:** normative statements decide a preference or a moral position on how the world should be. For example, we may say that monopolies are bad because its not *fair* for one company to make all the money.
- **Positive:** positive statements describe an objective or neutral point of view of the world. Monopolies generate a net social cost due to the distortionary wedge caused by inflated prices. Monopolies therefore decrease social welfare and are inefficient in markets.

[a] see the FTC guidelines and definition of monopolization here: https://www.justice.gov/archives/atr/monopoly-power-and-market-power-antitrust-law. Note that there is significant evidence that needs to be provided in order to prove an individual has either become or attempted to become a monopoly.

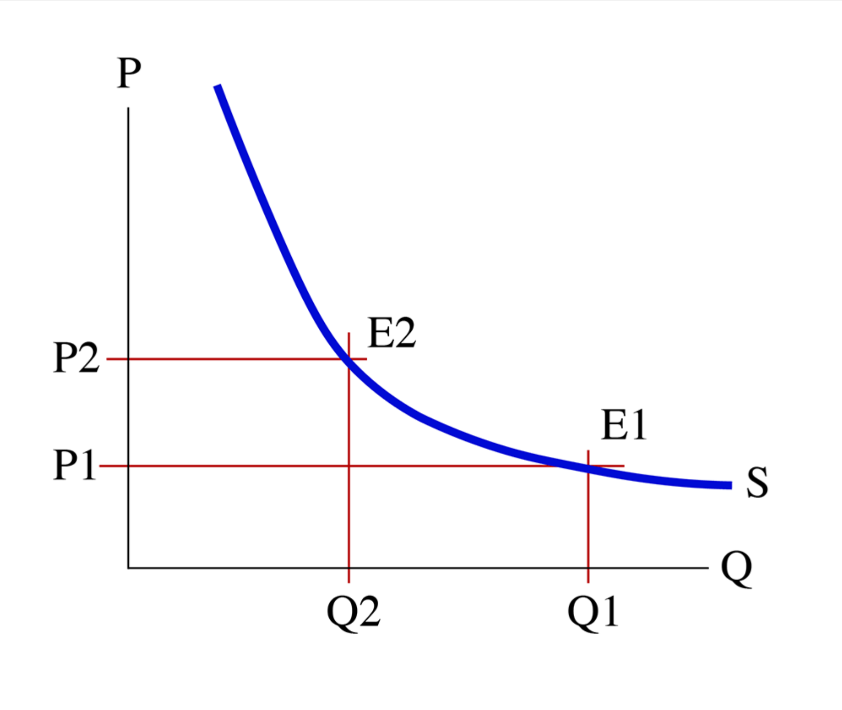
[b] Caremark Rx, Zinc Health Services, et al., In the Matter of (Insulin) accessible at: https://www.ftc.gov/legal-library/browse/cases-proceedings/221-0114-caremark-rx-zinc-health-services-et-al-matter-insulin

[c] https://www.ftc.gov/news-events/news/press-releases/2025/01/ftc-releases-summary-key-accomplishments

Solution 1.2: What are natural monopolies?

Natural monopolies exist in the somewhat rare cases where the marginal cost, that is, the cost of satisfying one more customer, is lower than the cost of servicing the previous customer. The marginal cost curve, and as such, the supply curve, is downward sloping or flat over the relevant range of production.

This is called we call a **natural monopoly** because it is economically efficient for there to only be one supplier.



Some examples of natural monopolies exist in markets in which there is a natural economy of scale: i.e. utilities and roads. Does it make sense for you - an individual - to build your own transmission line that connects to a power plant? Probably not, as the up front cost of capital is extremely high, especially for one single connection. The government allows utility companies (energy infrastructure providers) to have a natural monopoly so they may take advantage of the *economies of scale* and offer a lower price for consumers.

Other natural monopolies can be legally sanctioned by the government. The United States Post Office (USPS) is a legal monopoly, in that they have the sole legal authority to operate throughout the US. Yes, there are other distributors of mail, but only USPS is legally allowed to distribute first class mail.[a]

[a] https://psu.pb.unizin.org/introductiontomicroeconomics/chapter/chapter-8-monopoly/

Problem 2

Solution 2: Solving the monopolist problem

We know that the monopoly has cost $c(y) = 5y$ and faces a demand curve given by

$$D(p) = 100 - 4p.$$

(1) Its problem is given by

$$\max_{y} \ \pi = \left(25 - \frac{1}{4}y\right) y - 5y.$$

The FOC for the maximization problem is

$$\frac{\partial \pi}{\partial y} = 25 - \frac{1}{2}y^* - 5 = 0$$

Thus, $y^* = 40$ and $p^* = 15$

(2) The elasticity of demand at the optimal production level is

$$\varepsilon(y^*) = -4\frac{15}{40} = -1.5$$

(3) Its profits are

$$\pi = 15 \times 40 - 5 \times 40 = 400$$

Problem 3

Solution 3: Price discrimination

1. The monopolist's profit function is

$$\pi = (40 - y_1)y_1 + \left(30 - \tfrac{1}{2}y_2\right) y_2 - 10(y_1 + y_2).$$

   The maximization problem is

$$\max_{y_1, y_2} (40 - y_1)y_1 + \left(30 - \tfrac{1}{2}y_2\right) y_2 - 10(y_1 + y_2).$$

2. First-order conditions require

$$p_i(y_i) + p_i'(y_i)y_i = MC.$$

   Group 1:

$$(40 - y_1) - y_1 = 10 \quad \Rightarrow \quad 40 - 2y_1 = 10 \quad \Rightarrow \quad y_1^* = 15.$$

   Group 2:

$$\left(30 - \tfrac{1}{2}y_2\right) - \tfrac{1}{2}y_2 = 10 \quad \Rightarrow \quad 30 - y_2 = 10 \quad \Rightarrow \quad y_2^* = 20.$$

3. Optimal prices:

$$p_1^* = 40 - y_1^* = 25, \qquad p_2^* = 30 - \tfrac{1}{2}y_2^* = 20.$$

4. Elasticities:

$$\varepsilon_1 = \frac{p_1}{y_1} \cdot \frac{1}{p_1'} = \frac{25}{15} \cdot \frac{1}{-1} = -\frac{5}{3}.$$
$$\varepsilon_2 = \frac{p_2}{y_2} \cdot \frac{1}{p_2'} = \frac{20}{20} \cdot \frac{1}{-1/2} = -2.$$

   Group 1 has the lower elasticity in absolute value ($|\varepsilon_1| = 1.67 < 2$), meaning it is less price sensitive. Therefore group 1 pays the higher price, consistent with third-degree price discrimination.

5. Profit:

$$\pi = p_1^* y_1^* + p_2^* y_2^* - 10(y_1^* + y_2^*) = 25(15) + 20(20) - 10(35) = 375 + 400 - 350 = 425.$$

Week 9: solutions
ECON 100A

TA Allegra Saggese
Date November 24, 2025

Professor: Natalia Lazzati
Topic: Markets - review

Exercise problems are sourced primarily from Varian textbook and course materials (e-book), and from ECON204A (graduate microeconomics) with changes to the numerics developed with ChatGPT.

Problem 0

Solution 0.1: Perfect competition

A market is perfectly competitive when individual buyers and sellers are price takers—no single participant has any influence on the market price. This requires:

1. many buyers and sellers, each small relative to the market
2. identical (homogeneous, non-differentiated) products
3. full information (no asymmetries)
4. no barriers to entry, with free entry and exit

Under these conditions, firms take the market price as given and choose output to maximize profit where price equals marginal cost.

Perfectly competitive markets are preferred or *desirable* because they maximize total surplus (consumer + producer surplus).

Solution 0.2: Monopoly

We say a market is a monopoly when there is only one firm. A monopoly market is of interest such that a single firm has power to set the market price.

Recall that a monopoly may occur where there is barrier to entry, significant market power (by one firm), anti-competitive practices or strategies, and firms set prices. In practice, we say this may occur **naturally** when a firm has economies of scale - i.e. it is better for welfare maximization such that one firm can produce a good given it costs a lot to produce on a small-scale and that cost declines as quantity increases. Monopolies may occur without government support or allowance where a firm owns a scarce resource (i.e. one quarry that has all of the minerals or rock to produce), the firm imposes an entry cost against new firms in the market, or the firm files for a patent to give exclusive rights of production to that firm.
In a monopoly problem, we can measure the following:

1. Market power: $\frac{p(y^*)}{MC(y^*)}$
2. Deadweight loss: The difference between the willingness to pay of the consumers and the cost of production for all units that the monopolist should be producing but is not - i.e. $PS - CS = DWL$
3. Price discrimination: Calculate the supply for each group y_i, and determine who pays the highest price, based on elasticity, ϵ_i

Solution 0.3: Oligopoly

An oligopoly is an industry with a few firms that behave strategically. At the moment of making a decision, the firms need to anticipate what the other firms will do. This is related to **game theory**.

1. Cournot competition: Each of the firms selects how much to produce in order to maximize profits, taking as given the decision of the other firm. At equilibrium the predictions of the firms are correct. In Cournot, **firms compete on quantities.**
2. Stackleberg: A model of a leader and a follower. There are only two firms in the market, firms 1 and 2. Firm 1 is the leader and firm 2 is the follower. Firm 1 chooses first how much to produce. Firm 2 observes the decision of firm 1 and then selects how much to produce (quantity).
3. Bertrand competition: **Firms compete on prices**. Note that competition is tougher when firms compete in prices than when they choose quantities.

Solution 1.1: Short run competitive equilibrium

A short-run competitive equilibrium is a price–quantity pair in which:

- firms maximize profits taking the price as given,
- consumers maximize utility subject to their budget constraints,
- markets clear, and
- some inputs or the number of firms are fixed.

Mathematically, each firm chooses output q_i such that

$$p = MC_i(q_i),$$

and market clearing requires

$$\sum_i q_i = D(p)$$

Solution 1.2: Long run competitive equilibrium

A long-run competitive equilibrium is a price–quantity pair in which

1. firms maximize profits,
2. consumers maximize utility,
3. markets clear, and
4. all inputs and the number of firms can adjust freely.

Free entry and exit drive profits to zero.
Mathematically, the equilibrium satisfies

$$p = MC(q) = AC(q),$$

together with market clearing,

$$\sum_i q_i = D(p)$$

Solution 2: Perfectly competitive market solution

1. **Equilibrium**

   Equilibrium occurs where quantity demanded equals quantity supplied:

   $$30 - p = 2p.$$

   Solving,

   $$30 = 3p \quad \Rightarrow \quad p^* = 10,$$
   $$Q^* = 2p^* = 20.$$

2. **Diagram**

   A standard supply–demand diagram would show the downward-sloping demand curve $Q_d = 30 - p$, the upward-sloping supply curve $Q_s = 2p$, and the intersection at $(Q^*, p^*) = (20, 10)$.

3. **Consumer Surplus**

   Consumer surplus is the area of the triangle between the demand curve and the equilibrium price:

   $$CS = \frac{(30 - p^*)Q^*}{2} = \frac{(30 - 10)(20)}{2} = 200.$$

4. **Producer Surplus**

   Producer surplus is the area of the triangle between the supply curve and the equilibrium price:

   $$PS = \frac{(p^* - 0)Q^*}{2} = \frac{(10 - 0)(20)}{2} = 100.$$

5. **Total Surplus**

   $$TS = CS + PS = 200 + 100 = 300.$$

Solution 3: Monopoly market solution

We know the monopoly has cost $c(y) = 5y$ and faces the inverse demand curve

$$p(y) = 25 - \tfrac{1}{4}y,$$

since $D(p) = 100 - 4p \Rightarrow y = 100 - 4p \Rightarrow p = 25 - \tfrac{1}{4}y$.

Optimal output price: where the profit function is

$$\pi(y) = p(y)\, y - c(y) = \left(25 - \tfrac{1}{4}y\right) y - 5y.$$

Write it explicitly:

$$\pi(y) = 25y - \tfrac{1}{4}y^2 - 5y = 20y - \tfrac{1}{4}y^2.$$

FOC:

$$\frac{\partial \pi}{\partial y} = 20 - \tfrac{1}{2}y = 0.$$

Solve:

$$\tfrac{1}{2}y = 20 \quad \Rightarrow \quad y^* = 40.$$

Substitute into inverse demand:

$$p^* = 25 - \tfrac{1}{4}(40) = 15.$$

Elasticity of demand at y^*
Demand curve: $y = 100 - 4p$
Compute elasticity:

$$\varepsilon = \frac{p}{y}\frac{dy}{dp}.$$

Since $dy/dp = -4$,

$$\varepsilon(y^*) = \frac{p^*}{y^*}(-4) = \frac{15}{40}(-4) = -1.5.$$

Profits at the optimum:

$$\pi(y^*) = p^*y^* - 5y^* = 15(40) - 5(40) = 600 - 200 = 400.$$

Solution 4: Oligopoly market solution

(1) Firms choose quantity to maximize profit. For firm 1, the problem is

$$\max_{Q_1} \pi_1 = P(Q_1 + Q_2)Q_1 - C(Q_1) = (16 - (Q_1 + Q_2))\, Q_1 - Q_1$$

The problem of firm 2 is similar:

$$\max_{Q_2} \pi_2 = (16 - (Q_1 + Q_2))\, Q_2 - Q_2$$

(2) The reaction curve is the profit-maximizing quantity written as a function of the other firm's choice.
For firm 1, the first-order condition is

$$\frac{\partial \pi_1}{\partial Q_1} = 16 - 2Q_1 - Q_2 - 1 = 0.$$

Thus,

$$Q_1^*(Q_2) = \frac{15 - Q_2}{2}.$$

Doing the same for firm 2, we get

$$Q_2^*(Q_1) = \frac{15 - Q_1}{2}$$

(3) At the Nash equilibrium, both firms choose quantities that are optimal given their prediction of the other's behavior, and these predictions are correct. Thus,

$$Q_1^* = \frac{15 - Q_2^*}{2}, \qquad Q_2^* = \frac{15 - Q_1^*}{2}$$

Solving this system gives

$$Q_1^* = Q_2^* = 5$$

Week 10: solutions
ECON 100A

TA Allegra Saggese
Date December 1, 2025

Professor: Natalia Lazzati
Topic: Final exam review

Exercise problems are sourced primarily from Varian textbook and course materials (e-book), and from previous ECON100A materials provided by the Professor. Some changes to the numerics were developed with ChatGPT.

Problem 0: Coordination

In Cournot competition, each firm chooses its own quantity Q_i while taking the rival's quantity as fixed. In Cournot competition, each firm chooses its own quantity Q_i while taking the rival's quantity as fixed.

Solution 0

We know that $C_1(Q_1) = 4Q_1$ and $C_2(Q_2) = 4Q_2$. In addition,

$$P(Q_1 + Q_2) = 20 - 2(Q_1 + Q_2)$$

(1) Cournot Equilibrium

The maximization problems for firms 1 and 2 are:

$$\max_{Q_1}\{\pi_1 = [20 - 2(Q_1 + Q_2)]Q_1 - 4Q_1\}$$

$$\max_{Q_2}\{\pi_2 = [20 - 2(Q_1 + Q_2)]Q_2 - 4Q_2\}$$

The first-order conditions are

$$\frac{\partial \pi_1}{\partial Q_1} = 20 - 4Q_1 - 2Q_2 - 4 = 0$$

$$\frac{\partial \pi_2}{\partial Q_2} = 20 - 4Q_2 - 2Q_1 - 4 = 0$$

Note that we know from the first order conditions, that the first derivative with respect to quantity for both firms will be zero (i.e. $\frac{\partial \pi_i}{\partial Q_i} = 0$). This is because it is the firm's best response function *(BR function).* In Cournot equilibrium, both firms simultaneously choose quantities that best respond to each other. Therefore, $Q_2 = BR_2(Q_1)$ and $Q_1 = BR_1(Q_2)$, and this gives the point of the equilibrium such that neither firm wishes to deviate from these quantities, meaning that it is the best for both of them.
Solving,

$$Q_1(Q_2) = 4 - \frac{1}{2}Q_2 \qquad Q_2(Q_1) = 4 - \frac{1}{2}Q_1$$

Solving the system gives

$$Q_1^* = Q_2^* = \frac{8}{3}$$

We then get price from combining both of the quantities together to get the *aggregate quantity in the market.* Then, if the firms collude, they choose the total quantity $Q = Q_1 + Q_2$ to maximize joint profit. This is then modeled as a monopoly. So, collusion is modeled as such because monopolists restrict output to raise price. while Cournot firms produce more because each firm does not internalize how its own output reduces the rival's profit. Therefore collusion yields higher prices and lower quantities.
Equilibrium price:

$$P^* = 20 - 2(Q_1^* + Q_2^*) = 20 - 2\left(\frac{16}{3}\right) = \frac{28}{3}$$

(2) Collusion (Monopoly Behavior)

If the firms coordinate strategies, they behave as a monopolist, choosing total output Q:

$$\max_{Q}\{\pi = [20 - 2Q]Q - 4Q\}$$

FOC:

$$\frac{\partial \pi}{\partial Q} = 20 - 4Q - 4 = 0$$

Thus,

2

$$Q^* = 4, \qquad P^* = 12$$

1 Problem 1: Cournot competition (duopoly)

Solution 1, part 1-3

1. **Profit function:**

   Total quantity is $Q = Q_1 + Q_2$ and price is: $P = 100 - Q_1 - Q_2$. so, profit of firm 1:

   $$\pi_1 = (100 - Q_1 - Q_2)Q_1 - 20Q_1$$

   And we have profit of firm 2:

   $$\pi_2 = (100 - Q_1 - Q_2)Q_2 - 20Q_2$$

2. **Derive each firm's reaction function.**

   Firm 1:

   $$\frac{\partial \pi_1}{\partial Q_1} = 100 - 2Q_1 - Q_2 - 20 = 0,$$

   $$80 - Q_2 = 2Q_1, \qquad Q_1(Q_2) = \frac{80 - Q_2}{2}$$

   Firm 2:

   $$\frac{\partial \pi_2}{\partial Q_2} = 100 - Q_1 - 2Q_2 - 20 = 0,$$

   $$80 - Q_1 = 2Q_2, \qquad Q_2(Q_1) = \frac{80 - Q_1}{2}$$

3. **Solve for the Cournot–Nash equilibrium quantities.**

   At equilibrium:

   $$Q_1 = \frac{80 - Q_2}{2}, \qquad Q_2 = \frac{80 - Q_1}{2}$$

   By symmetry we know that $Q_1 = Q_2 = Q$ Substitute:

   $$Q = \frac{80 - Q}{2} \quad \Rightarrow \quad 2Q = 80 - Q \quad \Rightarrow \quad 3Q = 80,$$

   $$Q = \frac{80}{3} \rightarrow Q_1^* = Q_2^* = \frac{80}{3}$$

Solution 1, part 4-5

4 **Compute the equilibrium market price.**

Total quantity:

$$Q = Q_1^* + Q_2^* = \frac{160}{3}$$

Price:

$$P^* = 100 - \frac{160}{3} = \frac{140}{3}$$

5 **Suppose the firms collude and act as a monopolist. What output maximizes joint profits?**

A collusive cartel maximizes:

$$\pi = PQ - C(Q) = (100 - Q)Q - 20Q$$

$$\pi = 100Q - Q^2 - 20Q = 80Q - Q^2$$

FOC:

$$\frac{d\pi}{dQ} = 80 - 2Q = 0 \quad \Rightarrow \quad Q^M = 40$$

Thus, the collusive (monopoly) output is:

$$Q^M = 40 \qquad P^M = 100 - 40 = 60$$