Introduction to Microeconomics (ECON1)

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Course notes: synthesis

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[1] *Notes and content were adapted from 2024-25 UCSC Economics PhD ECON204A-C notes, Hal Varian's Microeconomics textbook, and ECON100A notes*

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Contents

1 Math Review

internal notes

Please not that none of this math is obligatory, nor is it the core objective of the course for you to understand this. That being said, it may significantly supplement your learning, and being familiar with these concepts may help you learn or visualize principles of economics. Additionally, some of these concepts will allow you to quickly move through graphics or simple exercises in the course.

1.1 Algebra (and linear algebra)

Functions are used constantly in economics: demand function, production function, and utility function are three common examples.

Theorem 1.1 (Function) *A function is a rule that tells you one specific result for each possible input. The core concept of the function is that it allows you to change the numeric value of inputs, while keeping the rule on determining the output the same. This is because it is a description of the relationship between variables, where the value of one variable (dependent variable) is determined by the value of another variable (independent variable).*

Functions are often denoted with a letter, f, g and then list the inputs into that function, x, y. In a **univariate** function, we only have *one free variable*. In a **multivariate** function, we have *two free variables*. This is the core difference between univariable and multivariable calculus (see below).

It is helpful to think of what the variables are in economics, for example: *What will the cost of eggs be if I change the supply (quantity)?*. This question can be represented as a function, f, that takes in one variable, q, to model how changes in q effect the numerical value of the output (cost).

$$\begin{aligned} f(q) &\Rightarrow \text{function} \\ c &\Rightarrow \text{cost, or generally the output} \\ q &\Rightarrow \text{quantity} \end{aligned}$$

Putting it all together we maybe can model the relationship as $f(q) = q^2 - 8q \equiv c$. So when $q = 10$, or there are ten eggs in the market, then the cost is: $f(10) = 10^2 - 8(10) \Rightarrow 100 - 80 = 20$. So when there are only ten eggs in the market (low quantity), each egg is \$20! Super expensive eggs...

Other examples of functions are:

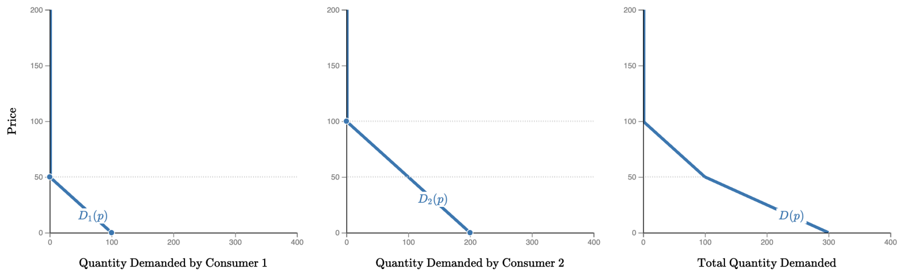
- Simple algebraic functions are still functions. For example, your monthly budget may be a function, comprised of the following: budget = textbooks + rent + groceries + cell phone bill, where each component can be changed.
- $y = mx + b \rightarrow$ this is a linear function, and this is the equation for the slope of the line. This is often used in economics to represent many different economic processes (i.e. supply and and demand). An example of this is the supply function, often represented in an equation like this: $Q_s = 15 + 2P$
- Weighted averages, such as the formula used to calculate your final grades, it may look something like this: $g = .25m + .25h + .1p + .4f$ where
  - g is the final course grade
  - m represents the midterm grade
  - h is the homework grade
  - p is the participation grade
  - f is the final exam grade

  Putting it all together, this function is a *weighted average*, and shows you how your grade is a composite of a number of different scores.

Theorem 1.2 (Line graphs) *Economics often uses line graphs, or graphical representations of a linear function. These graphs show a relationship between two variables: one measured on the horizontal axis (think horizon where the sun sets, so it is the one running along the bottom) and the other measured on the vertical axis. Each point on the graph represents a pair of values, and the points are connected to show the relationship clearly.*

Recall that line graphs make use of the linear relationship, $y = mx + b$ but other relationships, or functions, (non-linear) may be modeled on a graph. For example, a demand curve is drawn as a line graph where the price is on the vertical axis and quantity is on the horizontal axis. Each point shows how much consumers are willing to buy at a given price, and the line shows how quantity demanded changes as price changes.

Demand curves are often linear. Below is an example of demand curves, and then the summation of two individual consumers' demand curves together:



Theorem 1.3 (Growth rates) *A growth rate is the percentage change in some quantity over time. These are fundamental in economic problems, and it requires knowing the initial and ending quantity (or price, or whatever we measure the growth of).*

$$\text{Percent change} = \frac{Q_2 - Q_1}{Q_2}$$

$$\text{where } Q_1 \rightarrow \text{initial quantity}$$
$$\text{where } Q_2 \rightarrow \text{ending quantity}$$

Theorem 1.4 (Fundamental Theorem of Linear Algebra) *Relates the rank to the dimensions of the null space. We see that* $\dim(\mathit{Null}(A)) = \mathit{number\ of\ columns} - \mathit{rank}(A)$. *If you know how to solve (or which systems have) a solution, then when* $A\mathbf{x} = \theta$, *the solution set will be an affine subspace, where the dimensions are equal to the number of variables (columns) less the rank. This is the same as the FTLA where we see that* $\dim = \mathit{cols} - \mathit{rank}$.

Linear functions

- These are just mappings from the real space of a certain dimension to another. The functions preserve the vector space structure. We can also call it a linear transformation.
- **Theorem 1.5 (Linear equations)** *There is a vector* $\mathbf{a}$ *in* $\mathbb{R}^n$ *such that* $f(\mathbf{x}) = \mathbf{a}^T\mathbf{x}$ *for all* $\mathbf{x}$ *within* $\mathbb{R}^n$ *if* f *is a linear function. The proof demonstrates that if* $n = 3$, *and we have the basis of* $\mathbb{R}^3$ *in terms of vectors, then we can take any vector* $\mathbf{x}$*'s* (x_1, x_2, x_3) *and multiply it by the bases to get a linear equation. You can then see that this linear combination can be converted to a transpose of the function times the* $\mathbf{x}$ *vector.*
- **Theorem 1.6 (Solution when functions are linear)** *Similarly shows that if* f *is a linear function, there exists* $f(\mathbf{x}) = A\mathbf{x}$ *for all* $\mathbf{x}$ *values in* $\mathbb{R}^2$.

1.2 Calculus

- **Commutative laws**

- $A + B = B + A$

- **Distributive laws**
  - $A(B + C) = AB + AC$
  - $(A + B)C = AC + BC$
  - For multiplication, $ab = ba$ but for matrices it is **not** true that $AB = BA$

- **Associative laws**
  - $(A + B) + C = A + (B + C)$
  - $(AB)C = A(BC)$

- **Transpose**
  - Obtained by interchanging rows and columns: $A_{ij} \to A_{ji} = A^T$
  - Rules for the transpose:
    * $(A + B)^T = A^T + B^T$
    * $(A - B)^T = A^T - B^T$
    * $(A^T)^T = A$
    * $(rA)^T = rA^T$
    * $(AB)^T = B^T A^T$ (proof required)

- **Special matrices**
  - Square ($k = n$, same number of rows and columns)
  - Diagonal ($k = n$, and $a_{ij} = 0$ where $i \neq j$)
  - Identity: ones on the diagonal, and zeros elsewhere (i.e. $a_{ij} = 1$ where $a_{ij} = a_{jj}$)
  - Lower (and upper) triangular matrices: all entries below (or above) the diagonal are equal to zero ($a_{ij} = 0$ for all $a_{ij} \neq a_{jj}$)
  - Symmetric: $A^T = A$, i.e. $a_{ij} = a_{ji}$ for all i, j ($|A| = |A^T|$)
  - Idempotent: a square matrix B for which $B^2 = B$ (such as $B = I$)
  - Nonsingular matrix: a square matrix whose rank equals the number of its rows (or columns). If this is a matrix of coefficients in a system of linear equations, the system has *one and only one* solution.

- **Determinants**
  - Useful for defining if a system of linear equations has a solution (chapters 22, 24), for computing a solution where it exists (chapters 11, 14, 22, 24), and for determining whether a given nonlinear system can be well approximated with a linear one (chapter 13, specifically the implicit function theorem).
  - Useful properties:
    * The determinant does not change when you add a multiple of one row to another within A
    * Determinant changes sign when you interchange two rows of A: $|B| = -|A|$
    * If matrix $B = rA$, then $|B| = r \cdot |A|$
    * If two rows of A are the same, then the determinant is zero
    * If A is lower/upper triangular, the determinant is the product of the principal diagonal
    * If R is in row echelon form (REF), then $|R| = \pm|A|$
  - *Example cases:* If you have n linearly dependent vectors of order n, then the determinant is zero. If a square matrix of order n has linearly dependent columns, the inverse doesn't exist (det $= 0$ means no inverse).
  - **Theorem 1**: The determinant is defined as a sum of its minors, $a_{ik} \cdot (-1)^{i+j}$. From this, we can prove that the transpose determinant is the same as the determinant of the original matrix A.

- **Theorem 3**: A square matrix A is nonsingular if and only if $|A| \neq 0$. This is true because in REF, A would need to have all diagonal elements non-zero.
- **Theorem 4**: For square matrices, $|AB| = |A| \cdot |B|$

- **Inverse matrices**
  - Purpose: we use inverse matrices in absence of division. Where the inverse is $BA = I = AB$, we define the inverse of A as A^{-1}.
  - The inverse must be (a) square, (b) have the same dimensions as A.
  - $|A^{-1}A| = |I|$, $|A^{-1}||A| = 1$, where $|A| \neq 0$ (otherwise, the inverse does not exist).
  - If A, B are square invertible matrices:
    * $(A^{-1})^{-1} = A$
    * $(A^T)^{-1} = (A^{-1})^T$
    * AB is invertible, and $(AB)^{-1} = B^{-1}A^{-1}$
    * For any scalar r, rA is invertible and $(rA)^{-1} = \frac{1}{r}A^{-1}$
    * $A^rA^s = A^{r+s}$
  - **Theorem 5**: A^{-1} is unique. Prove this with the identity matrix or by showing the inverse exists by being $\frac{1}{\det(A)}$ times the transpose of the cofactor of A. Proof by contradiction can also show that if two matrices are both inverses, they must be the same by multiplication properties.

Systems of linear equations, *see an example below*

$$\begin{bmatrix} a_1 + 3a_2 \\ 2a_1 + 7a_2 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = X\mathbf{a}.$$

This is true in general. If

$$X = \begin{bmatrix} \mathbf{x}_1 & \mathbf{x}_2 & \dots & \mathbf{x}_n \end{bmatrix} \text{ and } \mathbf{a}^T = (a_1, a_2, ..., a_n)$$

then the linear combination in (8) is a vector that can be written as

$$a_1\mathbf{x}_1 + a_2\mathbf{x}_2 + ... + a_n\mathbf{x}_n = \textstyle\sum_{i=1}^n a_i\mathbf{x}_i = X\mathbf{a}.$$

Theorem 1.7 (Matrix rank and solution) *If A is an $m \times n$ matrix, then $A\mathbf{x} = \theta$ has a solution for a particular value of θ, if and only if (IFF) θ is in col(A). While $A\mathbf{x} = \theta$ has a solution for every single θ iffthe rank is equal to the number of rows (m).*

- **Consistent:** When a system has a solution (when solution is nonzero – affine subspaces).
- **Homogenous:** $A\mathbf{x} = 0$, meaning that the solution set of a homogenous system with n variables is zero and is a subspace of $\mathbb{R}^n$.
  - **Null space:** This is the subspace of solutions in the homogenous system, i.e., the value of zero (this is a subspace).

Building off Theorem 1.1,

Theorem 1.8 *If $\mathbf{d}$ is a solution in $\mathbb{R}^n$ of the system $A\mathbf{x} = \theta$ ($m \times n$ system of linear equations), then every other solution of $\mathbf{x}$ can be written as $\mathbf{x} = \mathbf{d} + \mathbf{v}$ (where $\mathbf{v}$ is in the nullspace(A)). Proof involves setting the definition of null space and the consistent system $A\mathbf{x} = \theta$ up.*

Definition of a derivative

- Let $f : \mathbb{R} \to \mathbb{R}$ be a function. The **derivative of f at a point** $x = a$, denoted $f'(a)$, is defined as:

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

  provided this limit exists.

- The derivative represents the *instantaneous rate of change* of f at the point a.
- If f is differentiable at each point in its domain, then it is differentiable.
- The differentiability of multivariate functions looks similar, except for h is a vector (normalized) and the second term is not just $f(x)$, but is $f(x) + a^T h$. If f is differentiable here, then we can say the derivative is $\mathbb{R}^n$ onto a_x.
- In the most general case where $f : \mathbb{R}^n \to \mathbb{R}^m$, we can create a very general formula for the differentiability of these larger functions.

Below are the set of rules that you can use to take the first derivatives. Note that these are given with a univariate case (one variable), but you can apply the same rules to partial derivatives in a multivariate case, treating the other variable as a constant.

Rule	**Formula**
Constant Rule	$\frac{d}{dx}[c] = 0$
Power Rule	$\frac{d}{dx}[x^n] = nx^{n-1}$
Constant Multiple Rule	$\frac{d}{dx}[c \cdot f(x)] = c \cdot f'(x)$
Sum Rule	$\frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$
Difference Rule	$\frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$
Product Rule	$\frac{d}{dx}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$
Quotient Rule	$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$
Chain Rule	$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$

Optimization:
Constrained optimization is essential to economics. For more information, review UChicago 15 page review on constrained optimization, accessible here https://home.uchicago.edu/~vlima/courses/econ201/pricetext/chapter2.pdf.

Definition 1.1 (Constrained optimization) *Constrained optimization studies maxima or minima of a function subject to restrictions on the variables. Examples include portfolio choice with a budget constraint, aircraft design limited by cost or weight, or a hiker whose path is constrained to a trail.*

Formally, the problem is

$$\max_{x \in \mathbb{R}^n} f(x) \quad s.t. \quad g_i(x) = 0 \ (i = 1, \dots, m), \quad h_j(x) \le 0 \ (j = 1, \dots, p).$$

Equality and inequality constraints reduce the feasible set. Instead of eliminating variables, the method of Lagrange multipliers reformulates the problem by adding multipliers so that first-order conditions describe the optimum.[2]

Theorem 1.9 *Suppose that k is an arbitrary constant and that f and g are differentiable functions at $x = x_0$. Then,*

$$\text{(a)} \quad (f \pm g)'(x_0) = f'(x_0) \pm g'(x_0),$$

$$\text{(b)} \quad (kf)'(x_0) = k\big(f'(x_0)\big),$$

$$\text{(c)} \quad (f \cdot g)'(x_0) = f'(x_0)g(x_0) + f(x_0)g'(x_0),$$

$$\text{(d)} \quad \left(\frac{f}{g}\right)'(x_0) = \frac{f'(x_0)g(x_0) - f(x_0)g'(x_0)}{g(x_0)^2},$$

$$\text{(e)} \quad \big((f(x))^n\big)' = n(f(x))^{n-1} \cdot f'(x),$$

$$\text{(f)} \quad (x^k)' = kx^{k-1}.$$

[2] Adapted from the definition provided by https://math.gmu.edu/~rsachs/math215/textbook/Math215Ch3Sec7.pdf

Theorem 1.2: *local minimum and maximum conditions*

(a) If $f'(x_0) = 0$ and $f''(x_0) < 0$, then x_0 is a maximum of f;

(b) If $f'(x_0) = 0$ and $f''(x_0) > 0$, then x_0 is a minimum of f; and

(c) If $f'(x_0) = 0$ and $f''(x_0) = 0$, then x_0 can be a maximum, a minimum, or neither.

2 Appendix

Table of mathematical notation

Symbol(s)	Explanation / Used For	Example
$\varnothing$ (Empty set)	Set with no element	$\lvert\varnothing\rvert = 0$
$\forall$ (For all)	Symbol to indicate all values in a set	$f(x) \geq f(x^*)\ \forall\ x \in \mathbb{R}$
$\mathbb{N}$ (N)	Set of natural numbers	$\forall x, y \in \mathbb{N},\ x + y \in \mathbb{N}$
$\mathbb{Z}$ (Z)	Set of integers	$\mathbb{N} \subseteq \mathbb{Z}$
$\mathbb{Z}_+$ (Z-plus)	Set of positive integers	$3 \in \mathbb{Z}_+$
$\mathbb{Q}$ (Q)	Set of rational numbers	$\sqrt{2} \notin \mathbb{Q}$
$\mathbb{R}$ (R)	Set of real numbers	$\mathbb{R} = (-\infty, \infty)$
$\mathbb{R}_+$ (R-plus)	Set of positive real numbers	$\forall x, y \in \mathbb{R}_+,\ xy \in \mathbb{R}_+$
$\mathbb{C}$ (C)	Set of complex numbers	$\exists z \in \mathbb{C}\ (z^2 + 1 = 0)$
$\mathbb{Z}_n$ (Z-n)	Set of integers modulo n	In $\mathbb{Z}_2$, $1 + 1 = 0$
$\mathbb{R}^3$ (R-three)	Three-dimensional Euclidean space	$(5, 1, 2) \in \mathbb{R}^3$
$f(x), g(x, y), h(z)$	Functions	$f(2) = g(3, 1) + 5$
a_n, b_n, c_n	Sequences	$a_n = \frac{3}{n+2}$
$h, \Delta x$	Limiting variables in derivatives	$\lim_{h\to 0} \frac{e^h - e^0}{h} = 1$
δ, ε	Small quantities in proofs involving limits	$\forall \varepsilon > 0,\ \exists \delta > 0$ such that $\lvert x\rvert < \delta \implies \lvert 2x\rvert < \varepsilon$
$F(x), G(x)$	Antiderivatives	$F(x)' = f(x)$
$\text{dom}(f)$	Domain of f	If $g(x) = \ln x$, then $\text{dom}(g) = \mathbb{R}_+$
$\text{ran}(f)$	Range of f	If $h(y) = \sin y$, then $\text{ran}(h) = [-1, 1]$
$f(x)$	Image of element x under f	$g(5) = g(4) + 3$
$f(X)$	Image of set X under f	$f(A \cap B) \subseteq f(A) \cap f(B)$
$f \circ g$	Composite function	If $g(3) = 5$, $f(5) = 8$, then $(f \circ g)(3) = 8$
$\sum_{i=m}^{n} a_i$	Sum of a_i	$\sum_{i=1}^{5} i^2 = 55$
$\prod_{i=m}^{n} a_i$	Product of a_i	$\prod_{i=1}^{n} i = n!$
$\bar{A}, A^c$	Complement of set A	$\bar{\bar{A}} = A$
$A \cap B$	Intersection of sets	$\{2, 5\} \cap \{1, 3\} = \varnothing$
$A \cup B$	Union of sets	$\mathbb{Z} \cup \mathbb{N} = \mathbb{Z}$
$A \setminus B$	Difference of sets	In general, $A - B \neq B - A$
$A \times B$	Cartesian product of sets	$(11, -35) \in \mathbb{N} \times \mathbb{Z}$
$\mathcal{P}(A)$	Power set of A	$\mathcal{P}(\varnothing) = \{\varnothing\}$
$\lvert A\rvert$	Cardinality of A	$\lvert\mathbb{N}\rvert = \aleph_0$
$\lVert v\rVert$	Norm of vector v	$\lVert(3, 4)\rVert = 5$
$u \cdot v$	Dot product	$u \cdot u = \lVert u\rVert^2$
$u \times v$	Cross product	$u \times u = 0$
$\lim_{n\to\infty} a_n$	Limit of sequence	$\lim_{n\to\infty} \frac{n+3}{2n} = \frac{1}{2}$
$\lim_{x\to c} f(x)$	Limit of function	$\lim_{x\to 3} \frac{\pi \sin x}{2} = \frac{\pi}{2} \lim_{x\to 3} \sin x$
$\sup(A)$	Supremum of A	$\sup([-3, 5]) = 5$
$\inf(A)$	Infimum of A	If $B = \{1, \frac{1}{2}, \dots\}$, then $\inf(B) = 0$
$f', f'', f''', f^{(n)}$	Derivatives	$(\sin x)''' = -\cos(x)$
$\int_a^b f(x)\,dx$	Definite integral	$\int_0^1 \frac{1}{1+x^2}\,dx = \frac{\pi}{4}$
$\int f(x)\,dx$	Indefinite integral	$\int \ln x\,dx = x \ln x - x + C$
$\frac{\partial f}{\partial x}$	Partial derivative	If $f(x, y) = x^2 y^3$, then $\frac{\partial f}{\partial x} = 2xy^3$
$a \in A$	a is an element of A	$\frac{2}{3} \in \mathbb{R}$
$a \notin A$	a is not an element of A	$\pi \notin \mathbb{Q}$
$A \subseteq B$	A is a subset of B	$A \cap B \subseteq A$
$A = B$	A is equal to B	If $A = B$, then $A \subseteq B$

Introduction to Microeconomics (ECON1)

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Professor KC Fung

[1] *Notes and content were adapted from Lazzati UCSC ECON100A notes, Fung UCSC ECON1-1 notes, Krugman & Wells Principle of Microeconomics textbook, and econgraphs.org*

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Contents

1 The market economy

Definition 1.1 (Competitive market) *A competitive market is a model, where we have many buyers and sellers. Buyers and sellers are producing or consuming the exact same good. No one individual person can influence the price at which the good is sold, and the competition within the market is what determines the price.*

Five key elements of a competitive market:

1. **the demand curve**: a line that traces how much of a good consumers will demand (*quantity*) at different prices. The demand curve is a graphical representation of the demand schedule.
2. **the supply curve**: the quantity of a good that is supplied (by producers) at different prices, represented again by a curve. It is generally upward sloping.

3a. factors that shift the demand curve

- changes in the price of substitutes or complements (other goods)
- changes in income
- changes in taste (preferences)
- changes in expectation: expectation of a future price, future changes in your personal income
- changes in the aggregate (total) number of consumers

3b. factors that shift the supply curve

- changes in input prices (labor cost, oil prices). This often shifts the supply curve. It shifts *towards the origin* if costs increase, and shifts *away from the origin* if the input price decreases, such as with the development of a cost saving technology
- change in the price of other goods or services
- change in technology (outward shift)
- change in expectations: such as anticipated price of inputs
- changes in the aggregate (total) number of producers

4. **the market equilibrium**: this is represented as the intersection of the supply and demand curves. In Figure 1 below, we see that there is no equilibrium price is *lower* than the price we set in the market artificially. At the equilibrium price, quantity demanded, Q_d is equal to quantity supplied, Q_s. When price > equilibrium price (as is below in Figure 1), there is a **surplus**. Sellers decrease the price. If price < equilibrium price, there is a **shortage**. Sellers will raise the price.
5. the change in the market equilibrium from a demand or supply curve shift: It is important to understand what happens to the equilibrium price and quantity when there is a shift of the supply curve, demand curve, or both. In simultaneous shocks (both supply and demand curve shift) the change in the market equilibrium will depend on the relative size and direction of both shifts.

Definition 1.2 (Substitutes) *A substitute good is a good that is purchased in place of another good. The relationship between the demand for these two goods is inverse, such that you either buy one good or the other. You do not buy both goods in increasing quantities. For example coffee and tea are often substitutes, so when the price of coffee rises, the demand for coffee decreases. Coffee's substitute, tea, did not have a change in price. Because of this, the demand for tea increases.*

Definition 1.3 (Complements) *A complement, or complementary good, is a good that is consumed (or purchased) alongside another good. For example, shoes and socks are often complements. When you purchase a pair of shoes, you may also need a pair of socks. Therefore, when the price of shoes rise, the quantity demanded for shoes decreases. This means the quantity demanded for socks also decreases.*

Definition 1.4 (Law of demand) *When the price of a good increases, the quantity demanded decreases. There is an inverse relationship between price and quantity demanded, for most goods. This is represented as a downward sloping demand curve on a graph where the y axis is the price, and the x axis is the quantity, and the demand curve starts with a high price and low quantity, and moves towards a higher quantity demanded as the price decreases.*

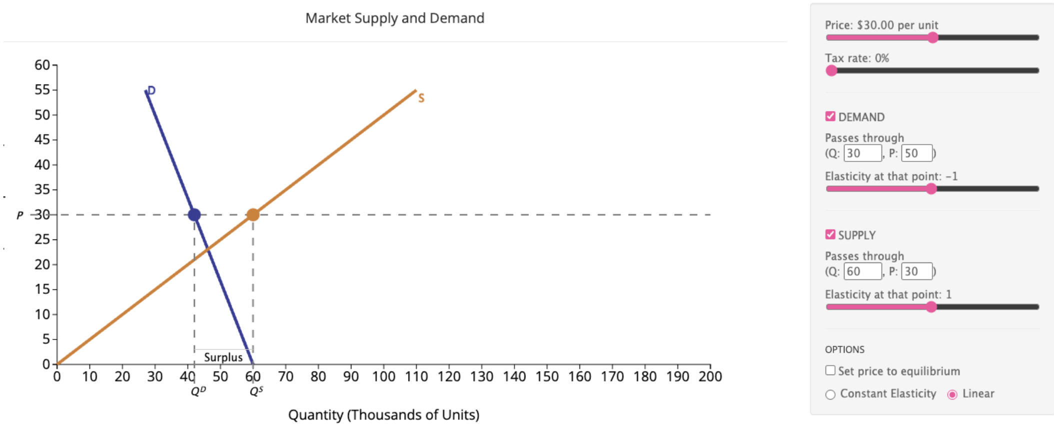


Figure 1: Supply and demand graph with surplus

2 Consumers

Economics has been called the dismal science because it studies the most fundamental of all problems, scarcity. Because of scarcity we all face the dismal reality that there are limits to what we can do. No matter how productive we become, we can never accomplish and enjoy as much as we would like. The only thing we can do without limit is desire more. Because of scarcity, every time we do one thing we necessarily have to forgo doing something else desirable. So there is an opportunity cost to everything we do, and that cost is expressed in terms of the most valuable alternative.[2] This idea should lead us to our idea of opportunity cost.

Definition 2.1 (Opportunity cost) *The opportunity cost is the value of one good (that you may consume) in terms of a second good. When we look at the production possibility frontier, we calculate the opportunity cost of one good, in terms of the other good. The slope of the PPF gives us the opportunity cost, i.e. how many pairs of shoes can we make (or buy) for how many slices of pizza. If we can purchase either four slices of pizzas or one pair of shoes, than the opportunity cost of shoes (in terms of pizza slices) is four.*

For a consumer, the slope of the budget line, $-\frac{p_1}{p_2}$ *is the opportunity cost of good 1. This is the relative trade-off between purchasing good 1 and spending your money on the alternative consumption. We have to give up some unit of good 1 in order to have the opportunity to consume good 2. This is the opportunity cost that the consumer faces.*

Definition 2.2 (Goods) *We assume that goods and services are homogeneous commodities, with a finite number of them. The commodity vector (or consumption bundle) is denoted by* $x = [x_1, \ldots, x_L]'$*, where each element represents a quantity of a specific good in* $\mathbb{R}^L$*, the commodity space.*

There are certain types of goods, which describe how a consumer sees these different goods, in relation to their income, their preference relative to other goods, or with respect to price. Below are some *types of goods* that can help explain the way preferences for goods will change over time. These classifications are not always mutually exclusive.

- **Neutral good**: A good in which the consumer is indifferent towards, regardless of the quantity. This implies that a neutral good sits on a vertical indifference curve. A neutral good can be a good (desire more, like another pair of cool sneakers) or a bad (desire less, like pollution).
- **Discrete good**: A good that is only available to be purchased in integer amounts (i.e. you can only buy one car, you cannot buy a part, or a fraction of the car).
- **Related goods (or services)**: These are goods that are substitutes or complements in production. Therefore, the price of related goods will effect the supply curve of a certain good. The producer's

[2] Adapted from Opportunities and Costs, by Dwight Lee. *The Freeman.*

supply curve is affected by whether the related good changes in prices. For example, higher oil prices lead to increased supply of natural gas because natural gas is a complementary related good, and if a producer is extracting more oil because demand increases, then it will also extract more natural gas.

- **Giffen good**: Goods where $\frac{dx}{dp} \geq 0$. This indicates that when price for the good increases, demand for the good also increases. This may apply to goods which are valuable, or rare, where demand increases with prices.
- **Normal good**: This is a typical good, where demand increases in income, or $\frac{dx}{dy} \geq 0$. As income rises, an individual will demand more of the normal good. Therefore, an increase in income leads to an outward shift in the demand curve (a shift to the right) for that normal good.
- **Inferior good**: This is a good that is purchases *less* when income increases. Inferior goods are decreasing in income, where $\frac{dx}{dy} \leq 0$. When income increases, the demand for inferior goods decreases, so the demand curve would shift inward (or to the left on the graph). [3]

Definition 2.3 (Consumer surplus) *The area that is bounded by the demand curve, the Y axis (price), and the actual price (the horizontal line running from the actual price) that the consumer pays. Consumer surplus is positive when the actual price consumers pay is low.*

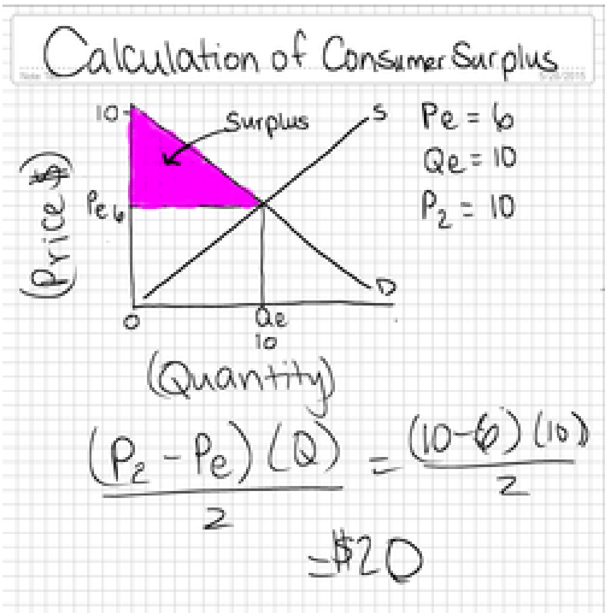


Figure 2: Consumer surplus (purple area)

Consumer surplus is easy to calculate. It requires knowing the formula for the area of the triangle as well as identifying the following information:

- Market price (along the y axis)
- The highest price demanded by consumers (where the Q_d line intersects the Y axis)
- Q_d at the market price (the x axis point, or the quantity, where the demand curve intersects the market price)

The consumer surplus can be calculated by taking these three pieces of information, and plugging into the area of a triangle formula. The formula is $\frac{1}{2}h \cdot b$ where h is the height of the triangle and b is the

[3] It's important to note you cannot have a utility function dependent on x_i goods, where $i = 1, ...n$ where all goods are inferior.

base (or the length) of the triangle. Ignore the hypotenuse.

$$
\begin{aligned}
&= \frac{1}{2} h \cdot b \quad \rightarrow \text{plug in values from Fig 2} \\
&= \frac{1}{2}(10 - p_e) \cdot (Q_e - 0) \\
&= \frac{1}{2}(10 - 6) \cdot (10 - 0) \\
&= \frac{1}{2}(4) \cdot (10) \\
&= 2 \cdot 10 \\
&= 20
\end{aligned}
$$

The formula used above is slightly different than the one presented in Figure 2. Both are correct, but the steps above should demonstrate how you can take the area of a triangle formula, and plug in the values from the graph for the calculation. The figure provides a quick trick by using Q_e without subtracting zero.

Definition 2.4 (Utility) *Utility is not a numeric measure of happiness. Instead, utility is a measure of consumer preference. Utility is a way for us to rank the preferences of a consumer.*

Definition 2.5 (Marginal utility) *The rate of change (or tradeoff) between a consumer's preference for one good over another is called the marginal utility. It is a ratio between the rate of change in utility, $\triangle U$ and the rate of change of the good of interest (in this case good 1) - $\triangle x_1$. So we use the following formulas:*

$$MU_1 = \frac{\triangle U}{\triangle x_1} \qquad MU_2 = \frac{\triangle U}{\triangle x_2} \qquad \triangle U = MU_2 \triangle x_2$$

Marginal utility changes with respect to the form of the utility function. Therefore, choice behavior only tells us about rankings of preferences, and not about the marginal utility.

Definition 2.6 (Marginal rate of substitution) *A utility function is used to measure the slope of the indifference curve at a given bundle of goods. MRS therefore is the slope at any given bundle - or the amount a consumer is willing to swap of good 1 for a bit of good 2. We use the formula:*

$$MRS = \frac{\triangle x_2}{\triangle x_1} = -\frac{MU_1}{MU_2}$$

3 Producers

The lowest price at which a seller may sell is their cost. When sellers sell a good *at cost* they make no profit. An individual producer may get a surplus - calculated as the difference between the actual price received and the cost they paid to produce the good. Producer surplus, generally, is calculated as the aggregate (or sum) of individual surpluses from all sellers in a market.

Definition 3.1 (Producer surplus) *The producer surplus is the triangular area, underneath the market price, but above the supply curve. The area is bounded by the supply curve, the Y-axis, and the market price that producers all receive for selling in the competitive market.*

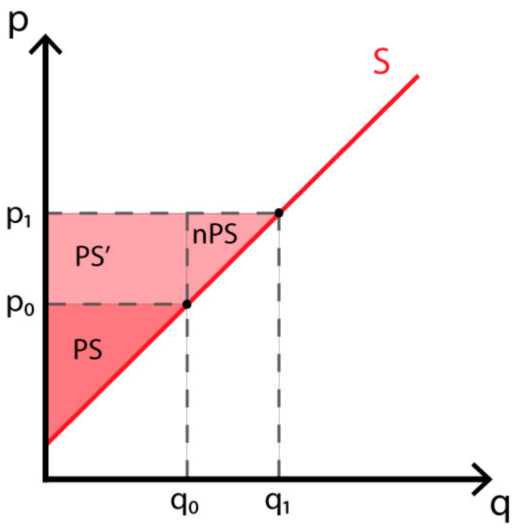


Figure 3: Producer surplus

4 Welfare

Policymakers, or what we often refer to as the *social planner* in economics, is the person or are the set of people that step in when the market's invisible hand seems to fail. [4] When the market fails and the social planner steps in to make adjustments, they often are identifying a simple failure that can be corrected. Do both buyers and sellers have full information on the good being provided? Is there free entry into the market, so all sellers can compete? This is intervention to ensure markets function. **But what about long run economic growth?**

Sometimes the social planner wants to support the long run objective of growing the economy and making people better off. This is good, not just for the social planner himself to be complemented, and potentially awarded in future elections, but also to improve the economic outcomes of all individuals, and thus bringing gains to individuals across society. This decision is often known as the *equity-efficiency tradeoff*. Taken from Sachs (2017)[5]:

> *A market economy, it is said, is efficient: national income is maximized as profit-maximizing businesses and utility-maximizing consumers meet in the marketplace. The resulting market equilibrium may, however, be inequitable (unfair), with an excessive gap in income between the rich and poor. Taxing the rich to give money to the poor can then raise equity (fairness) but at the cost of distorting market incentives (such as incentives for hard work) and thereby lowering national income. The proverbial pie is shared more equally but the pie is smaller.*

Definition 4.1 (Equity-efficiency tradeoff) *In welfare economics, when the social planner attempts to improve the Pareto efficiency of an economy, they must balance the potential gains in both efficiency (distortion free, optimal allocation) and equity (fairness in allocation). In a market economy, we believe there exists a tradeoff between growing the pie and increasing every person's share. This creates a tradeoff, where transfers and taxation that are used to correct the unequal distributions in market outcomes may potentially lead to distortions in people's incentives to work or consume.*

Putting it all together, particularly the concepts of consumer surplus and producer surplus, we can better understand the market economy, and the outcomes they may produce. Firstly, we will learn the positives:

- gains from trade in the market
- market efficiency

[4] Recall the **invisible hand**, as described by Adam Smith, was a simple metaphor to explain how individuals in markets often made choices that were optimal, or efficient for the aggregate market.

[5] https://static1.squarespace.com/static/5d59c0bdfff8290001f869d1/t/5f2b00a3d499fe2ca12c649e/1596653732105/Sachs2017_Chapter_TheEfficiency-EquityTradeoff.pdf

- property rights and markets
- prices as economic signals

Next we will learn about some market issues, that can be measured in terms of surpluses:

- equity issues (as addressed in the equity-efficiency tradeoff)
- market failures (such as monopolies, negative externalities, or private information)
- public good allocation

Definition 4.2 (Deadweight loss) *Total loss in surplus which occurs when an action or policy reduces the total quantity of goods (or services) transacted in a market. This reduction in quantity is below the efficient market equilibrium quantity.*

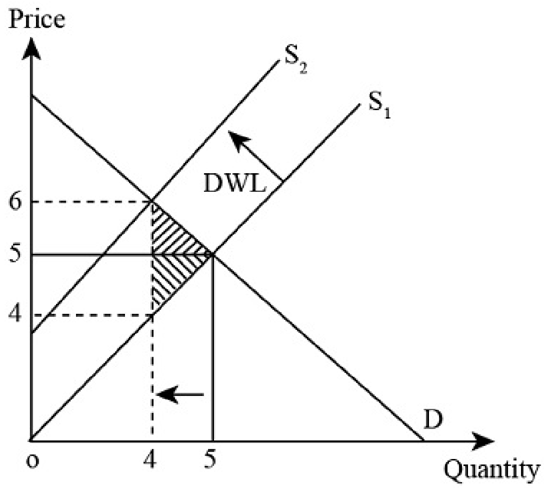


Figure 4: Deadweight loss from a shift in the supply curve

5 Price controls

What are **price controls**? Price controls are instruments that a government or regulator can use to control the price within a market. These are government-imposed legal restrictions on what prices a good can be sold or bought for in a market.

Price controls can be ceilings (such as rent control, or a cap on the price so it cannot be increased) or floors (such as a minimum wage, or price support of corn so that the price never falls to zero). Price controls override the competitive market's ability to find its own equilibrium, and instead do not allow Q_s and Q_d to adjust naturally, as a response to price. A price control matters only if it prevents the market from reaching the competitive equilibrium price.

Definition 5.1 (Binding price controls) *A price control is* ***binding*** *if it forces the market price away from the competitive equilibrium price. This results in inefficiencies (such as inefficiently low quantities)*

Let p^* denote the competitive equilibrium price. A price control is binding when it is set so that the market cannot reach p^*.

- A price ceiling is binding if $p_c < p^*$.
- A price floor is binding if $p_f > p^*$

Binding price controls change the quantity traded and reduce total surplus by creating deadweight loss. Binding controls result in the following effects:

- Binding price ceiling ⇒ shortage
- Binding price floor ⇒ surplus

Definition 5.2 (Non-binding price controls) *A price control is* ***non-binding*** *if it does not affect the market outcome because the equilibrium price already satisfies the law. This type of price control does not constrain market behavior. A price ceiling is non-binding if* $p_c \geq p^*$ *A price floor is non-binding if* $p_f \leq p^*$. *In the non-binding case, there is no shortage or surplus, and consumer and producer surplus remain unchanged.*

Real world examples of price controls are:

Policy	**Control**	**Equilibrium**	**Binding?**
Rent control	$p_c = 1000$	$p^* = 1500$	Yes
Rent control	$p_c = 2000$	$p^* = 1500$	No
Minimum wage	$p_f = 15$	$p^* = 12$	Yes
Minimum wage	$p_f = 9$	$p^* = 12$	No

Intuition: why use price controls? Price ceilings are implemented when there is inefficiently low quantities supplied in the market or inefficient allocation of sales among the sellers. Additionally, price floors are implemented when there is little surplus available to producers, or wasted resources.
There are also **quotas**, which are a quantity instrument. These set limits on the quantity of a good available for purchase or available to be bought in a market. Quotas also create deadweight loss and **quota rents**: where earnings accrue to a license holder from the ownership of the right to sell the good.

6 Elasticities

Definition 6.1 (Price elasticity of demand) *Change in the quantity demanded,* Q_d *as a result in a one percentage point change in the price. The elasticity tells us how much does demand change if the price changes by 1%?*

Formula:

$$e = \frac{\%\text{ change } Q_d}{\%\text{ change } price}$$

To calculate using the **midpoint method**:

$$-\frac{(Q_2 - Q_1)/(Q_1 + Q_2)}{2} \Big/ \frac{(P_2 - P_1)/(P_1 + P_2)}{2}$$

where $(Q_1 + Q_2)/2$ is the midpoint

To calculate using the linear downward sloping demand curve, we can take the change in quantity and change in price against the slope, such that:

$$e = -\frac{\triangle Q}{Q} \Big/ \frac{\triangle P}{P} = \frac{\triangle Q}{\triangle P} \cdot \frac{P}{Q}$$

Elasticity can be interpreted as:

- $e > 1$: elastic (when demand is perfectly elastic ($e = \infty$), it is horizontal line on the price vs. quantity graph, like the center horizontal line in the capital letter E). When p increases by 1%, demand decreases by **more** than 1%.
- $e < 1$: inelastic (when demand is perfectly inelastic ($e = 0$), it is a vertical line on the price vs. quantity graph, line the shape of letter I). When p increases by 1%, demand decreases by **less** than 1%.
- $e = 1$: unit-elastic. When p increases by 1%, demand decreases by **exactly** than 1%.

Price elasticity is affected by:

1. the availability of substitutes. When there are many substitute goods, the price elasticity is very high - consumers respond to a change in the price.

2. the type of good. When the good is luxury it is often quite elastic. When a good is a basic necessity (electricity, toilet paper), the good is often low or inelastic.

3. total income: when consumption is a large share of income, consumption tends to be elastic.

Other properties of price elasticity of demand include the relationship between total revenue TR and elasticity. We know $TR = price \cdot quantity$, so when price increases a little, quantity may decrease a lot or little, depending on the elasticity. For example:

- $e > 1$: price $\uparrow$, quantity $\downarrow\downarrow$, so $TR \downarrow$
- $e < 1$: price $\uparrow$, quantity $\downarrow$, so $TR \uparrow$
- $e = 1$: price $\uparrow$ by 1%, quantity $\downarrow$ by 1%, so TR does not change

Definition 6.2 (Cross price elasticity) *The cross price elasticity is the relative change in demand for good a over a change in demand for good b. This captures the magnitude and direction of a change in demand for one good when the quantity demanded for another good changes by 1%.*

The formula:

$$E_{a,b} = \frac{\%\triangle Q_a}{\%\triangle Q_b}$$

This depends on the relationship between goods a and b:

- **Substitutes:** $E_{a,b} > 0$, or the elasticity is positive. This is because the increase in demand for good b will decrease the demand for good a. The consumer buys one or the other, not both.
- **Complements:** $E_{a,b} < 0$, or the elasticity is negative. If the price of hot dogs goes up, then the demand for hot dogs decreases. The demand for hot dog buns, purchased in a certain ratio with hot dogs, will therefore decrease as well.

Definition 6.3 (Income elasticity of demand) *This elasticity is the change in quantity demanded for a certain good when an individual's income increases by 1%.*

The formula:

$$E_I = \frac{\%\triangle Q_x}{\%\triangle \text{ income}}$$

This depends on the type of good the individual is buying:

- **Normal:** $E_I > 0 \rightarrow$ As income increases, the consumer buys more normal goods.
- **Inferior:** $E_I < 0 \rightarrow$ As income increases, the consumer buys less inferior goods.

Definition 6.4 (Price elasticity of supply) *This elasticity is the change in quantity supplied for a certain good when the price of the good changes by 1%.*

The formula:

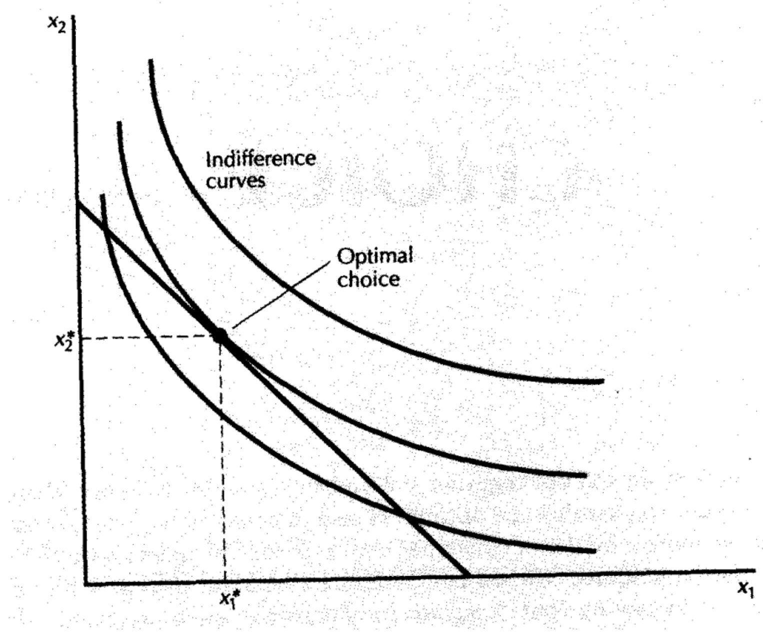
$$e_S = \frac{\%\triangle Q_s}{\%\triangle P_s}$$

Given that the supply curve is upward sloping, we know generally that e_S is positive, or greater than zero. This is because, as price goes up, the producer will likely supply more of the good. Elasticity of supply is often affected by the availability of inputs to producers.

- $e_S > 1 \rightarrow$ elastic
- $e_S = 1 \rightarrow$ unit-elastic
- $e_S < 1 \rightarrow$ inelastic

7 Taxes

Expanding on consumer choice, we can now say that *optimal choice* is where consumers choose the most preferred bundle from their budget sets.



In optimality (and seen in the above graph), we know that the consumers optimal bundle is written, in notation as (x_1^*, x_2^*). This point is where the graphic is indicating *optimal choice.*

Definition 7.1 (Taxes and subsidies) *Taxes are an additional cost on top of the price of the good the seller wants to sell it at. The government always imposes taxes. Taxes are in addition to the price,* p_1, p_2 *and can be applied in different ways. Subsidies instead are money given either to a producer or consumer to lessen the cost of a good. It is subtracted from the price.*

1. **Quantity tax:** The government imposes a per unit tax on a good. For example, Santa Cruz has a sugary beverage tax. For each ounce of a drink, there is a .02 cent tax. A tax introduces a per unit addition to the price, such that $p_1 \Rightarrow (p_1 + t)$.

2. **Value tax**: A value tax is applied on the price of a good, such that there is a percentage, added additional on the price, in which is a tax. For example, a 7.25% value tax is added to all goods in California. This sales tax is a value tax.

3. **Lump sum**: The government charges a set fee, regardless of purchases, budget, or income to every person. The opposite of a lump sum is a transfer.

4. **Subsidy**: A subsidy is the opposite of a tax, in which the government compensates the consumer. For example, a quantity subsidy is a discount per unit of a good purchased, such that prices goes from $p_1 \rightarrow (p_1 - s)$. Subsidies can also be added *ad valorem*, as a value subsidy.

Definition 7.2 (Income tax) *An income tax is a one-off taxation on individuals. It may result in differential welfare effects than a per-unit (quantity) tax. Income tax is non-distortionary, in that the entire budget set of the individual is shrunk, and the relative price of goods does not change. When the income tax is* ***uniform****, it is applied in the same way across all individuals.*